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ITERATIVE CONTROLLER DESIGN METHOD BASED ON CLOSED-LOOP IDENTIFICATION

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Abstract

This paper proposes an iterative design method of robust controllers which achieve the robust stability as well as the low sensitivity based on the closed loop identification. In this method we re-design the controller based on the knowledge of the closed loop modeling error which is obtained from the experiment. The effectiveness of the proposed method is shown by the experiment of positioning control of the flexible joint motor.

1 Introduction

Recently, much attention is paid to the joint design of control and identification. When we design a control system, for example, using H_∞ control theory, we must repeat re-design of controller many times by changing the criterion function to obtain the desirable closed loop property. Since this process involves the trial and error, it is important to establish the systematic method of this iteration.

The importance of the joint design of identification and control is pointed out [1], and to solve this issue, many systematic methods are proposed [2]~[7]. Zang and Gevers discussed a joint design of LQG control and least square identification [2] [3], and Schrama connected H_∞ control with closed loop identification, based on coprime factorization [7]. These methods are proposed mainly from the view point of identification, and try to minimize the criterion from both side of the identification and the controller design. To put it concretely, they identify the model in the closed loop and design the new controller based on the model and repeat this procedure. However there are some problems in these methods as follows. First, since the characteristics of closed loop system depends on both the model and the controller, if both of them are changed in every step, we can not expect the convergence of these methods [8]. Second, because the H_2 norm criterion is commonly adopted, it is difficult to take advantage of H_∞ control theory which is widely accepted in the robust control field. Third, there are few reports in the experimental verification.

Therefore, in this paper, we propose a new method of the combined identification and control which satisfies the followings : (i) The convergence of the iterative method is guaranteed, and (ii) we can take advantage of the H_∞ control methodology. In addition, we evaluate its effectiveness via experiments.

We consider SISO systems in this paper.

2 Problem Formulation

Consider the system shown in Fig.1, where P is a real plant, K is a controller and r is a reference input. First, we make some following assumptions.

[A.1] The plant P and its model P_{m0} are given, and the unstable zeros and the relative degree of P are known.

[A.2] The desired model G_m of closed loop system is given, whose unstable zeros include those of P.

[A.3] The controller K_0 is given which stabilizes both P and P_{m0}, and achieves the exact model matching[9] i.e.,

$$G_{m0}(P_{m0}, K_0) = G_m \tag{1}$$

where G_{m0} means the closed loop system shown in Fig.1 with $P = P_{m0}, K = K_0$, and K_0 has the following form.

$$u = K_0 \begin{bmatrix} r \\ y_p \end{bmatrix} \tag{2}$$

$$K_0 =: [\ K_{f0} + K_{b0}G_m \quad -K_{b0}\ ] \tag{3}$$

$$K_{f0} =: P_{m0}^{-1}G_m \tag{4}$$

$$K_{b0} \ : \ \text{stabilizing both } P \text{ and } P_{m0} \tag{5}$$

[A.4] The closed loop system $G_{p0}(P, K_0)$ constructed by P and K_0 satisfies

$$\lim_{s\to 0}\{G_{p0}(s) - G_m(s)\} \to 0 \tag{6}$$

in the presence of the plant uncertainty.

A.4 implies that K_{b0} has an integrator. In this paper, we aim at improving the characteristics of G_p. For this purpose, we iterate the controller design and the system identification of G_p, and design the controller $K = K_i$ $(i = 1, 2, 3...)$. Under the above assumptions, we choose the criterion function as

$$J_\infty = \|G_p(P, K) - G_m\|_\infty \tag{7}$$

In the following, we use the suffix i, which means the number of iteration. For example, the closed loop system constructed by P and the i-th controller K_i $(i = 1, 2, 3, \ldots)$ is described as $G_{pi}(P, K_i)$ and the criterion function which is achieved by G_{pi} is expressed as $J_{\infty i}$. By using these expressions, our purpose is to give a systematic controller design method of finding K_i $(i = 1, 2, 3, \ldots)$ which satisfies

$$J_{\infty i} < J_{\infty(i-1)}, \ (i = 1, 2, 3, \ \ldots) \tag{8}$$

From the view point of Two-degree-of-freedom control, it is equivalent to finding K_{fi} and K_{bi} as

$$K_i = \ [K_{fi} + K_{bi}G_m \quad -K_{bi}] \tag{9}$$

3 Controller design method

3.1 Definition of the model uncertainty

The plant uncertainty is usually represented by multiplication form, that is,

$$P = (1 + \Delta)P_m \tag{10}$$

or additional form,

$$P = P_m + \Delta \tag{11}$$

However in this paper, we define the uncertainty Δ between the closed loop system G_p and G_m as

$$G_{pi} = \Delta_i G_m \tag{12}$$

By using (12), Δ_i becomes a biproper and minimum phase system under **A.1** and **A.2**, which makes it easy to identify Δ_i. The criterion function given by (7) is equivalent to

$$J_{\infty i} = \|(\Delta_i - 1)G_m\|_\infty \tag{13}$$

3.2 Controller design procedure

Next we give the design procedure.

[Step 1] Using the step response of G_{p0} and G_m, identify the uncertainty Δ_0 in (12). Suppose that Δ_{ident} is obtained from the experimental data and it is represented as follows,

$$\Delta_{ident} = \Delta_0 + \hat{\Delta}_0 \tag{14}$$

Δ_0 means the nominal uncertainty between G_{p0} and G_m, and $\hat{\Delta}_0$ means the uncertainty of Δ_0. When we identify Δ_0 from the experimental data, the noise is often contained in high frequency. Therefore we regard the low frequency part of Δ_{ident} as Δ_0 and the high frequency part as $\hat{\Delta}_0$ (see, e.g., Fig.7).

[Step 2] Add a new controller K_1' on the outside of the closed loop system G_{p0} as shown in Fig.2, in order to decrease the criterion function (13).

(a).Decrease of the criterion function

The transfer function from r to y_{p1} in Fig.2 becomes

$$y_{p0} = \frac{1 + G_m K_1'}{1 + G_{p0}K_1'} G_{p0} r = \frac{1 + G_m K_1'}{1 + \Delta_0 G_m K_1'} \Delta_0 G_m r \tag{15}$$

and the criterion function in (13) becomes

$$J_{\infty 1} = \left\| \frac{(\Delta_0 - 1)G_m}{1 + \Delta_0 G_m K_1'} \right\|_\infty \tag{16}$$

So, in order for the inequality

$$J_{\infty 1} < J_{\infty 0} \tag{17}$$

to be satisfied,

$$\left\| \frac{(\Delta_0 - 1)G_m}{1 + \Delta_0 G_m K_1'} \right\|_\infty \cdot \frac{1}{\|(\Delta_0 - 1)G_m\|_\infty} < \beta_1 \tag{18}$$
$$(0 < \beta_1 < 1)$$

must hold. It means that the following inequality

$$J_{\infty 1} < \beta_1 \cdot J_{\infty 0}, \ (0 < \beta_1 < 1) \tag{19}$$

should be satisfied.

(b).Robust stabilization

Since the identified Δ_0 in Step 1 has an uncertainty caused by noise, we cannot obtain the exact value of Δ_0 in the high frequency. For this reason, the next inequality must be guaranteed and the stability of the closed loop system must be kept,

$$\left\| \frac{\Delta_0 G_m K_1'}{1 + \Delta_0 G_m K_1'} W_2 \right\|_\infty < 1 \tag{20}$$

where W_2 must be a proper and high pass frequency function.

(c).Low sensitivity

The sensitivity function of the actual plant P in Fig.2 is given by

$$\frac{1}{1 + PK_{b0}} \cdot \frac{1}{1 + \Delta_0 G_m K_1'} \tag{21}$$

To prevent the sensitivity from getting worse in the low frequency,

$$\left\| \frac{1}{1 + \Delta_0 G_m K_1'} \cdot \alpha_1 \cdot \hat{W}_3 \right\|_\infty < 1 \ \ (\alpha_1 \geq 1) \tag{22}$$

($\hat{W}_3$: Low pass function satisfying $\lim_{s \to 0} \hat{W}_3(s) = 1$)

must be satisfied.

From these observation, K_1' is designed from the generalized control system in Fig.3, and makes the H_∞ norm of the transfer function from w_1 to z less than 1 with,

$$W_1 = \frac{1}{\|(\Delta_0 - 1)G_m\|_\infty} \cdot \frac{1}{\beta_1} \quad (0 < \beta_1 < 1) \qquad (23)$$

$$W_3 = \alpha_1 \cdot \hat{W}_3 \quad (\alpha_1 \geq 1) \qquad (24)$$

The disturbance w_2 is added to satisfy the standard assumption of H_∞ generalized plant[10], therefore W_4 may be $W_4 \ll 1$. The transfer function from w_1 to z_1, z_2 and z_3 in Fig.3 correspond to (18), (20) and (22), respectively.

[Step 3] Add a feedforward controller as shown in Fig.4. By using Δ_0^{-1}, we can improve J_∞.

[Step 4] We obtain the actual controller K_1 by the following equation (see Fig.5),

$$K_1 = [\ K_{f1} + K_{b1}G_m \quad -K_{b1}\ ] \qquad (25)$$

$$K_{f1} =: K_{f0}\Delta_0^{-1} + (\Delta_0^{-1} - 1)K_{b0}G_m \qquad (26)$$

$$K_{b1} =: K_{f0}K_1' + (1 + G_m K_1')K_{b0} \qquad (27)$$

[Step 5] In this method, the dimension of K_1 is very large. So, we must reduce the order of controller. In case of the reduction, notice that **A.4** must be satisfied, i.e. the feedback part of K_1 must have a pole at origin.

[Step 6] Replace K_0 by K_1, and repeat the steps 1 ~ 4.

3.3 Discussions

3.3.1 Improvement of the criterion function

By the feedback controller, the criterion function in (13) in the n-th iteration becomes as follows with β_i in (18)

$$J_{\infty n} = \|G_{pn} - G_m\|_\infty \leq \prod_{i=1}^{n} \beta_i \times \|G_{p0} - G_m\|_\infty \qquad (28)$$

It is obvious that $J_{\infty n}$ decreases as n grows, because of $0 < \beta_i < 1$. It is the consequence of treating the closed loop system as an object for control. In addition, by using feedforward controller Δ_0^{-1}, since the criterion function becomes

$$J_{\infty 1} = \left\| \frac{(\Delta_0^{-1}\Delta_{real} - 1)G_m}{1 + \Delta_{real}G_m K_1'} \right\|_\infty \qquad (29)$$

Δ_{real} : Real uncertainty

if $\Delta_0 \simeq \Delta_{real}$, we can expect the more decrease of J_∞.

3.3.2 Existence of the H_∞ controller

In the Step 2 of the proposed design procedure, the solution K_1' always exists by appropriate choice of α_i in (24) and β_i in (23). In fact, if we choose $\alpha_i = \beta_i = 1$, we have the trivial solution $K_i' = 0$ and G_{pi} becomes stable because $G_{p(i-1)}$ is stable. Therefore all we have to do is to look for larger $\alpha_i (> 1)$ and smaller $\beta_i (< 1)$ such that the solution exists.

3.3.3 Advantage of the product expression (12)

According to the definition of Δ in (12), Δ is biproper and minimum phase under the assumptions **A.1** and **A.2**. This implies that we can identify both poles and zeros of Δ just by obtaining the gain characteristics. Therefore, utilizing the improved model ΔG_m, we can decrease the criterion via Figs.2 and 3. This is quite different from the usual situation when we use the representation (10) or (11).

3.3.4 Renewal of model P_m

The identification of Δ corresponds to identify the model P_m. In other words, the $(i+1)$-th model $P_{m(i+1)}$ is renewed by using P_{mi}, Δ_i, and K_{bi} as

$$P_{m(i+1)} = \left[P_{mi}^{-1}\Delta_i^{-1} + (\Delta_i^{-1} - 1)K_{bi}\right]^{-1} \qquad (30)$$

However, since $P_{m(i+1)}$ exists in the closed loop, we cannot obtain the correct value of $P_{m(i+1)}$ from (30) because of noise and so on. For example, in this method, since K_{bi} has a integrator then it is difficult to know the property of $P_{m(i+1)}$ in low frequency. For this reason, the proposed method regards $\Delta_i G_m$ as the object of identification to avoid these problem. From this observation, we can say that this method renews the plant model and guarantees the convergence of design procedure simultaneously.

4 Experimental evaluation

In this section, we apply the proposed method to the positioning control of the flexible joint motor and verify its effectiveness by experiment.

4.1 Modeling

The experimental system is shown in Fig.6. This system has the motor and the disk which has large inertia and they are combined with the flexible joint. The transfer function of this system is given by

$$y = \frac{3.30 \times 10^5}{s(s+13.3)(s-\omega_r)(s-\bar{\omega}_r)}\tau \qquad (31)$$

$$\omega_r = -9.13 + 58.2j$$

where the output y is the rotation angle of the disk and the input τ is the torque of the motor. This system has a resonance mode at 58.2(rad/sec). We got this model from the physical model of this system, regarding it as a mechanical system of mass, spring and damper. This motor has the generating capacity of 100(W) and its maximum output is 6.70(Nm).

4.2 Controller design

We set G_m as

$$G_m = \frac{45^4}{(s+45)^4} \quad (32)$$

and regard the expression of (31) as the model P_{m0}, and design K_0 from P_{m0} and G_m, whose dimension is as follows.

$$\dim(K_{f0}) = 4, \quad \dim(K_{b0}) = 5 \quad (33)$$

[Step 1] We identify Δ_0 from impulse response by the spectral analysis using FFT as shown in Fig.7.

$$\Delta_0 = \frac{0.709(s+a_1)(s+\bar{a}_1)(s+a_2)(s+\bar{a}_2)}{(s+b_1)(s+\bar{b}_1)(s+b_2)(s+\bar{b}_2)} \quad (34)$$

$$a_1 = -6.58 + 63.7j, \quad a_2 = -16.2 + 30.0j$$

$$b_1 = -1.51 + 33.9j, \quad b_2 = -12.7 + 52.7j$$

It contains a vibratile mode at 33.9 (rad/sec) which is different from the system resonance mode 58.2(rad/sec) in (31).

[Step 2] We set $\beta_1 = 0.1$ and W_2, W_3 as

$$W_2 = \frac{15000(s+10)^2}{(s+10000)^2}, \quad W_3 = \frac{30}{s+10} \quad (35)$$

and design K_1'.

[Step 3, 4, 5] We calculate the actual controller K_1 and reduced it as follows.

$$\dim(K_{f1}) = 7, \quad \dim(K_{b1}) = 11 \quad (36)$$

[Step 6] We repeat the steps 1~4, and get K_2. Then we set $\beta_2 = 0.45$ and dimension of K_2 is as follows.

$$\dim(K_{f2}) = 10, \quad \dim(K_{b2}) = 12 \quad (37)$$

The step responses of G_{p0} and G_{p2} are shown in Fig.8. The size of the step signal is 60 (degree). The obtained response y_{p2} is closer to y_m than y_{p0}.

4.3 Discussions

The value of the criterion function in each iteration is as follows.

$$J_{\infty 0} = \|G_{p0} - G_m\|_\infty = 4.22 \quad (38)$$

$$J_{\infty 1} = \|G_{p1} - G_m\|_\infty = 0.579 \quad (39)$$

$$J_{\infty 2} = \|G_{p2} - G_m\|_\infty = 0.457 \quad (40)$$

Since the H_∞ norm is very difficult to calculate precisely in the actual system, we calculate it by

$$J_{\infty i} = \|(\Delta_i - 1)G_m\|_\infty \quad (41)$$

(where Δ_i is the identified value in i-th iteration)

And there exists a resonance mode in the mechanical device which did not appeared in (31). This mode appears because of the existence of motor friction. We can cope with this mode by regarding G_p as a controlled object. And the responses of G_{pi}, $(i = 1,2,3)$ are shown in Fig.9, when 20(Nm) valued disturbance is added to motor. It is easy to see that the low sensitivity is achieved.

5 Conclusion

In this paper, we have proposed a new iterative controller design method based on closed loop identification, where we can highly expect the convergence of the method and can take advantage of the robust control methodology. In addition, we have shown its effectiveness by the experiment using the flexible joint motor.

References

[1] R. J. P. Schrama, "Accurate Identification for Control : The Necessity of an Iterative Scheme", *IEEE Transaction on Automatic Control*, **37**, No.7, pp.991 - 994 (1992)

[2] Z. Zang, R. R. Bitmead and M. Gevers, "H_2 Iterative Model Refinement and Control Robustness Enhancement", *30th IEEE Conf. Decision and Control*, pp.279 - 284 (1991)

[3] Z. Zang, R. R. Bitmead and M. Gevers, "Disturbance Rejection: On-line Refinement of Controllers by Closed Loop Modeling", *Proc. of ACC*, pp.2829 - 2833 (1992)

[4] K. Liu and R. Skelton, "Closed-Loop Identification and Iterative Controller Design", *29th IEEE Conf. Decision and Control*, pp.482 - 487 (1990)

[5] D. E. Rivera, J. F. Pollard, L. E. Sterman and C. E. Garcia, "An Industrial Perspective on Control-Relevant Identification", *Proc. of ACC*, pp.2406 - 2410 (1992)

[6] W. S. Lee, B. D. O. Anderson, R. L. Kosut and I. M. Y. Mareels, "On Adaptive Robust Control and Control-Relevant System Identification", *Proc. of ACC*, pp.2834 - 2841 (1992)

[7] R. J. P. Schrama and Paul M. J. Van den Hof, "An Iterative Scheme for Identification and Control Design Based on Coprime Factorization", *Proc. of ACC*, pp.2842 - 2846 (1992)

[8] M. Gevers, Towards a Joint Design of Identification and Control?, *Essays on Control, Trentelman, Willems Eds.,(Birkhäuser)*. pp.111 - 151 (1993)

[9] T. Sugie and T. Yoshikawa, "General Solution of Robust Tracking Problem in Two-Degree-of-Freedom Control Systems", *IEEE Transaction on Automatic Control*, **AC-31**, No.6, pp.552 - 554 (1986)

[10] K. Glover and J. C. Doyle, "A State Space Approach to H_∞ Optimal Control", *Three Decades of Mathematical System Theory, H.Nijmeijer, J.M. Schumacher Eds.*, pp.179 - 218 (1989)

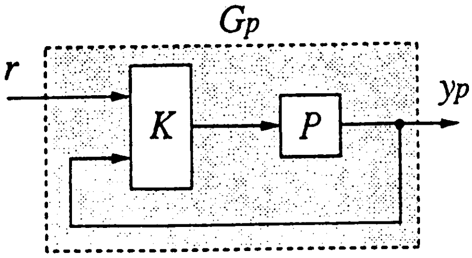

Fig.1 Closed loop system

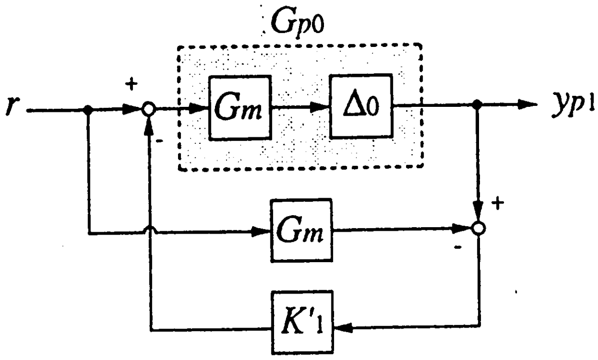

Fig.2 Closed loop system with K'1

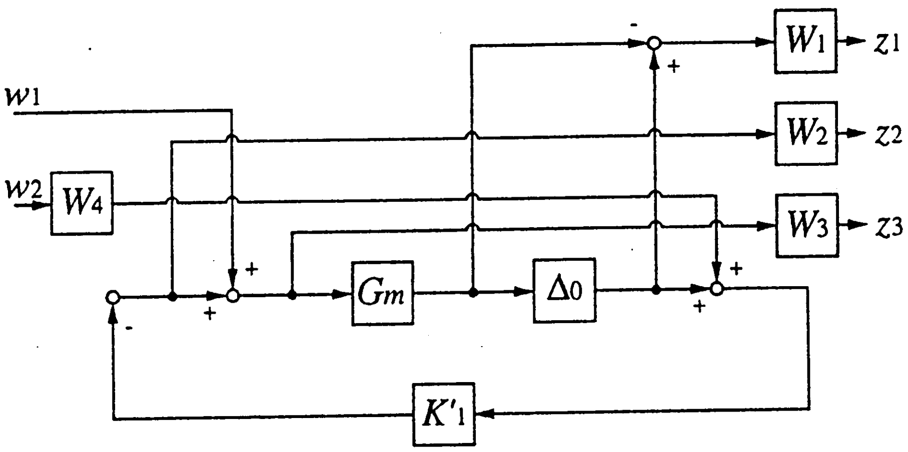

Fig.3 Generalized control system

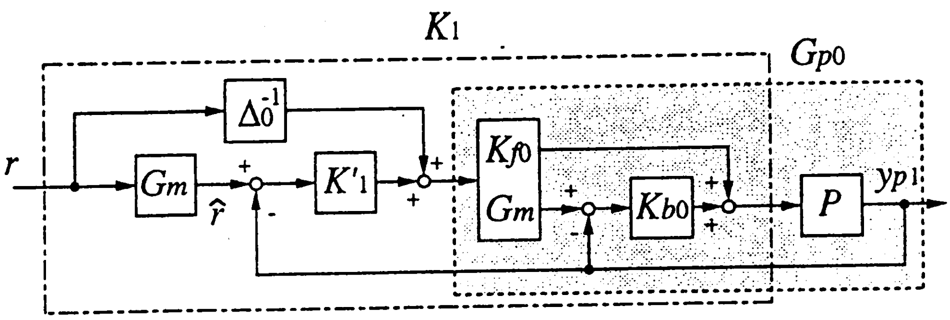

Fig.4 Design of feedforward controller

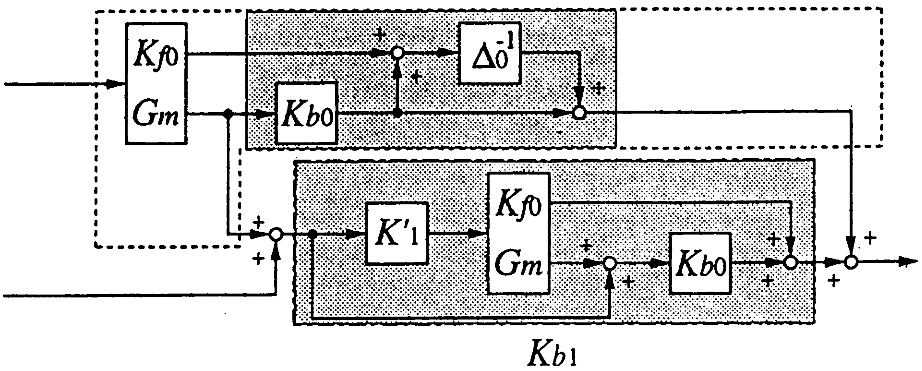

Fig.5 Achieved closd loop system

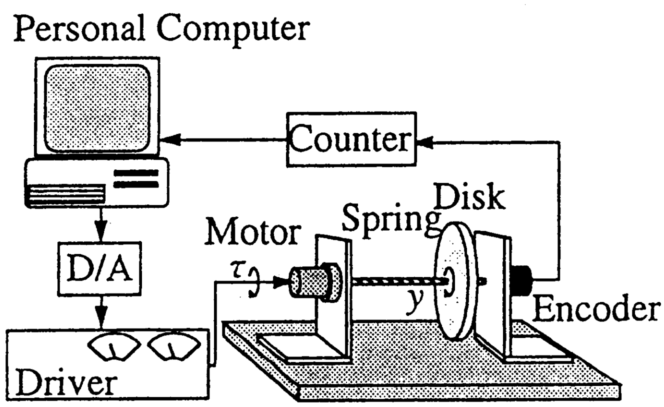

Fig.7 Experimental system

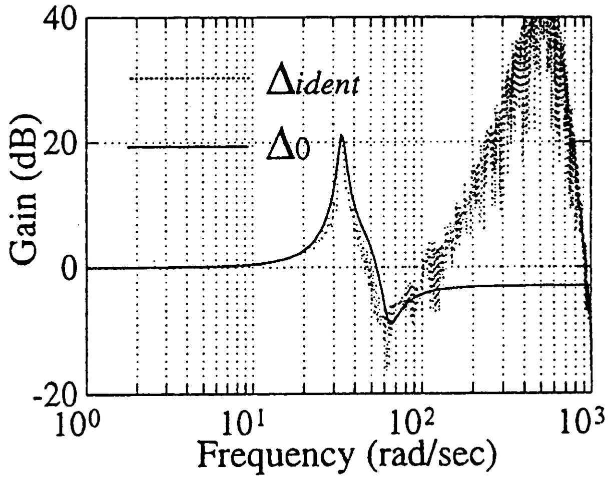

Fig.7 Identification of Δ_0 and $\hat{\Delta}_0$

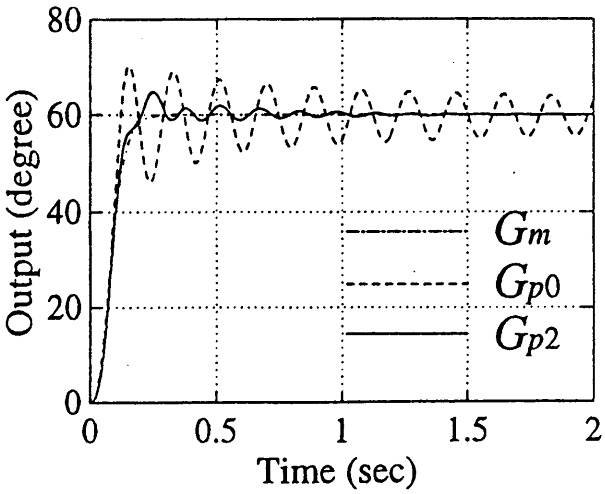

Fig.8 Step response of G_{p0} and G_{p2}

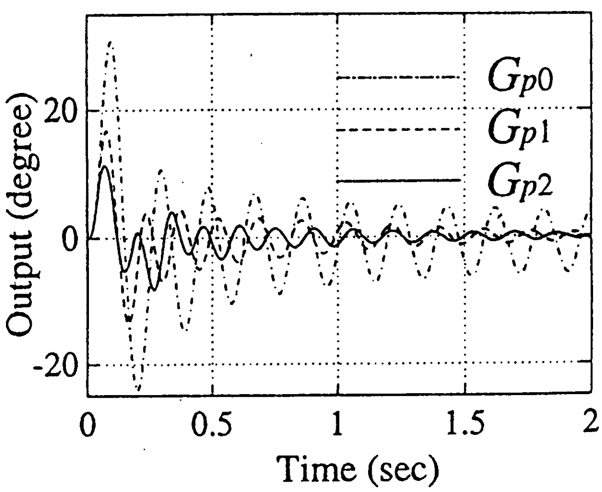

Fig.9 Disturbance rejection