Controller Reduction for Pseudo-Reference in High-Degree of Freedom Control System

Masaaki Watanabe*
Dept. of Mechanical Sciences and Engineering, Tokyo Institute of Technology, Tokyo, JAPAN

Masafumi Okada†
Dept. of Mechanical Sciences and Engineering, Tokyo Institute of Technology, Tokyo, JAPAN

Dong Dung Ngyuen‡
Dept. of Mechanical and Control Engineering, Tokyo Institute of Technology, Tokyo, JAPAN

ABSTRACT

Dance teaching, sports teaching and rehabilitation require instruction of motion from an expert to a beginner. So far, we have proposed "Pseudo-Reference" based on attractor design method. The pseudo-reference is a virtual reference which is derived from a controller of an autonomous control system and shows a target posture to execute the motion. However, for high-degree of freedom systems, because the controller has to be a high order function, it is not easy to be obtained. In this paper, we propose a controller reduction method for attractor design. Based on the correlation of motion data, the principal component analysis gives us appropriate low dimensional space of the motion. The effectiveness is evaluated by using the tap dancing robot, and pseudo-reference is applied to human motion.

Index Terms: I.2.9 [ARTIFICIAL INTELLIGENCE]: Robotics—Kinematics and dynamics; I.2.8 [ARTIFICIAL INTELLIGENCE]: Problem Solving, Control Methods, and Search—Control theory

1 INTRODUCTION

For human-human motion instruction, we sometimes use dance notation or time sequence posture variation. The dance notation was developed to hand down a traditional dance to posterity. However the dance notation is difficult to understand for beginners, because it is for experts who acquaint themselves with terpsichorean art. Moreover, for example, the time sequence posture of the long jump is illustrated in the textbook of gymnastics as shown in Figure 1. Because the figure represents only the kinematic information of the motion, some annotations are used to explain the dynamical characteristic of the motion. The instructor will say 'Jump like running up stairs' or 'Put your head forward'. We may imagine the corresponding postures, but it is difficult for beginners because the annotations include personal inspirations. The annotations frequently represent a knack of motion which is an important factor to make efficient motion. This concept is similar to human knowledge which consists of explicit and implicit knowledge[10]. For smooth communication, the appropriate representation of implicit knowledge plays an important role[4]. The kinematic postures and the athlete's annotations analogize with explicit and implicit knowledge respectively. From these considerations, the embodiment of the athlete's instinct with a posture data will contribute to the effective motion instruction.

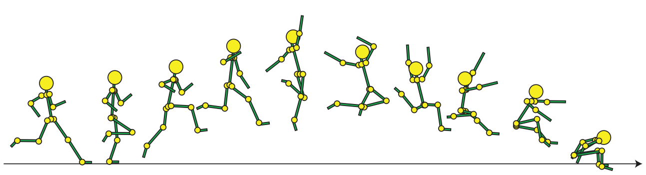

Figure 1: The time sequence posture of long jump

Some results with a same concept have been reported for robot control. Hasegawa et al.[13] and Cortesao et al.[11] divided human skill of peg-in-hole into three motions and they had the robot realize this task with appropriate selection of motion. Dordevic et al.[3] defined human skill from motion elements of experts. These methods focused on the representative motions to execute the given task effectively. Kuniyoshi et al.[15] showed a knack of motion to be obtained from a lot of measured data. Kawamura et al.[5] focused on the turning points of rotation, angular velocity and acceleration in the motion data, and defined another type of knack of motion. These methods give us an important motion key frame from dynamical point of view. In terms of annotation embodiment, we proposed a "Pseudo-reference"[6] based on modeling of human motion with autonomous controlled system. It gives us the timing and amplitude of the input torque which represents a knack of motion. In this method, the human motion is modeled by an autonomous controlled system based on orbit attractor[8], and the implicit reference is embodied as a pseudo-reference from a dynamical point of view.

However, the autonomous system is difficult to be obtained for a high-degree of freedom system because of the higher order controller. Some controller reduction methods have been reported for robot control. Moore[7] focused on model reduction of minimal realization for linear system by using principal component analysis. Villemagne et al.[2] focused on controller reduction by using canonical interaction analysis for linear system. Anderson et al.[1] discussed the many approaches for controller reduction with linear state-space equation. Though these methods are applied to controllers in linear system, it is difficult to apply to a nonlinear state feedback controller.

In this paper, we propose a reduction method of nonlinear controller in attractor design for high-degree of freedom system. Based on the high correlation between joint angle data of humans[12], a state-space projection is obtained based on principal component analysis. The effectiveness of the proposed method is evaluated by using a tap dancing robot, and pseudo-reference is applied to human walking motion in sagittal plane.

2 ATTRACTOR DESIGN AND PSEUDO-REFERENCE

2.1 Controller design based on orbit attractor

To obtain pseudo-reference, a given motion (for example, motion capture data) is modeled by an autonomous system[8] based on attractor design method[9]. In this section, we summarize the attractor design method simply. Consider the robot dynamics represented by the following nonlinear difference equation in discrete time do-

*e-mail: watanabe@micro.mep.titech.ac.jp
†e-mail: okada@mep.titech.ac.jp
‡e-mail: nguyen.d.aa@m.titech.ac.jp

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main;

$$\boldsymbol{x}[k+1] = f(\boldsymbol{x}[k]) + g(\boldsymbol{x}[k], \boldsymbol{u}[k]) \tag{1}$$

where $\boldsymbol{x} \in R^n$ is a state variable, $\boldsymbol{u} \in R^m$ is an input at time stamp k. The controller is designed by a nonlinear function of $\boldsymbol{x}$ as follows;

$$\boldsymbol{u}[k] = h(\boldsymbol{x}[k]) \tag{2}$$

so that $\boldsymbol{x}$ is entrained to a specified closed curved line Ξ as;

$$\Xi = \begin{bmatrix} \xi_1 & \xi_2 & \cdots & \xi_N \end{bmatrix} \quad (\xi_{N+1} = \xi_1) \tag{3}$$

Ξ is assumed to be realizable which means there exists an input sequence that realizes motion Ξ for the dynamics. The block diagram of the autonomous system using equation (1) and (2) is shown in Figure 2. The robot realizes the motion Ξ without external input.

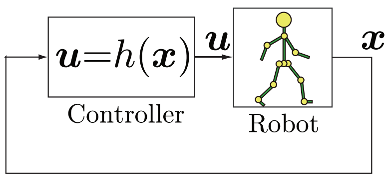

Figure 2: Autonomous control system

For a nonlinear function in equation (2), a ℓ-th order polynomial of $\boldsymbol{x}$ as;

$$\begin{aligned} \boldsymbol{u} &= a_0 + a_1\boldsymbol{x} + a_2\boldsymbol{x}^2 + \cdots + a_\ell \boldsymbol{x}^\ell && (4)\\ &= \Theta\phi(\boldsymbol{x}) && (5)\\ \Theta &= \begin{bmatrix} a_0 & a_1 & \cdots & a_\ell \end{bmatrix} && (6)\\ \phi(\boldsymbol{x}) &= \begin{bmatrix} 1 & \boldsymbol{x}^T & \boldsymbol{x}^{2T} & \cdots & \boldsymbol{x}^{\ell T} \end{bmatrix}^T && (7) \end{aligned}$$

is utilized. a_i $(i = 1, \cdots, \ell)$ and Θ are coefficient matrices of the polynomial function, and $\phi(\boldsymbol{x})$ expands $\boldsymbol{x}$ to the power vector. For example, $\boldsymbol{x} \in R^3$ and $j = 2$ defines $\boldsymbol{x}^j$ as;

$$\boldsymbol{x}^2 = \begin{bmatrix} x_1^2 & x_1x_2 & x_1x_3 & x_2^2 & x_2x_3 & x_3^2 \end{bmatrix}^T \tag{8}$$

By setting realizable pairs of $(\boldsymbol{u}_i, \boldsymbol{x}_i)$, Θ is obtained by the least mean square approximation as;

$$\begin{aligned} \Theta &= \boldsymbol{U}\Phi^{\#} && (9)\\ \boldsymbol{U} &= \begin{bmatrix} \boldsymbol{u}_1 & \boldsymbol{u}_2 & \cdots & \boldsymbol{u}_N \end{bmatrix} && (10)\\ \Phi &= \begin{bmatrix} \phi(\boldsymbol{x}_1) & \phi(\boldsymbol{x}_2) & \cdots & \phi(\boldsymbol{x}_N) \end{bmatrix} && (11) \end{aligned}$$

where $[\,\cdot\,]^{\#}$ means a pseudo inverse matrix defined by;

$$\Phi^{\#} = \Phi^T \left(\Phi\Phi^T\right)^{-1} \tag{12}$$

2.2 Pseudo-reference design

In this section, we summarize the pseudo-reference design method[6]. The closed loop system in Figure 2 does not have any external input. By using the decomposition of the controller, the system of Figure 2 is changed into that of Figure 3 which has a virtual reference $\boldsymbol{x}^{ref}$ inside the controller. By considering the comparison between autonomous system and linear controlled systems, $h(\boldsymbol{x})$ is decomposed. The state variable $\boldsymbol{x}$ converges to Ξ by the attractor design as $k \to \infty$. On the other hand, there are two methods which realize $\boldsymbol{x} = \xi$. One is a two degree of freedom control system (model matching) [14] as shown in Figure 4. P is a plant, P_m^{-1} is a inverse dynamical model of P, K is a feedback controller that

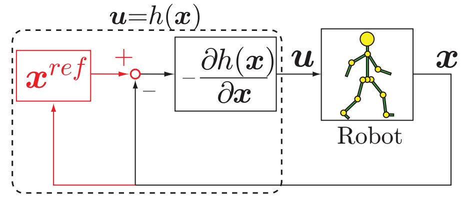

Figure 3: Robot control system using the pseudo-reference

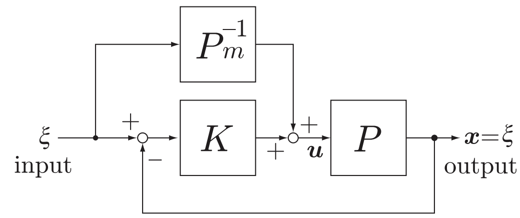

Figure 4: Two DOF model matching control system

stabilizes P and $\boldsymbol{u}$ is an input to P. Because the transfer function from the input to the output of the closed loop system is equal to 1 with the assumption $P_m = P$, $\boldsymbol{x}$ converges to ξ as $k \to \infty$. In this system, $\boldsymbol{u}$ is obtained by;

$$\boldsymbol{u} = P_m^{-1}\xi + K(\xi - \boldsymbol{x}) \tag{13}$$

The other method, which realizes $\boldsymbol{x} = \xi$, is appropriate selection of reference in the one degree of freedom control system as shown in Figure 5. K is the same feedback controller in Figure 4, and $\boldsymbol{x}^{ref}$ is the reference motion pattern. By setting $\boldsymbol{x}^{ref}$ as;

$$\boldsymbol{x}^{ref} = \frac{1 + PK}{PK}\xi \tag{14}$$

$\boldsymbol{x}$ converges to ξ as $k \to \infty$. In this system, $\boldsymbol{u}$ is represented as;

$$\boldsymbol{u} = K(\boldsymbol{x}^{ref} - \boldsymbol{x}) \tag{15}$$

The first order Taylor expansion of equation (2) around $\boldsymbol{x}$ using $\xi = \boldsymbol{x} + \delta$ $(\delta \ll 1)$ gives the following equation.

$$\boldsymbol{u} = h(\xi) - \frac{\partial h(\boldsymbol{x})}{\partial \boldsymbol{x}}(\xi - \boldsymbol{x}) \tag{16}$$

$\partial h/\partial \boldsymbol{x}$ is differential of $h(\boldsymbol{x})$ with respect to the each element of $\boldsymbol{x}$. For example, $\boldsymbol{x} \in R^3$ defines $\partial h/\partial \boldsymbol{x}$ as;

$$\frac{\partial h(\boldsymbol{x})}{\partial \boldsymbol{x}} = \begin{bmatrix} \frac{\partial h(\boldsymbol{x})}{\partial x_1} & \frac{\partial h(\boldsymbol{x})}{\partial x_2} & \frac{\partial h(\boldsymbol{x})}{\partial x_3} \end{bmatrix}^T \tag{17}$$

By comparing equation (13) and (16), because the first term is concerned to ξ and the second term is concerned to $\xi - \boldsymbol{x}$, we can regard

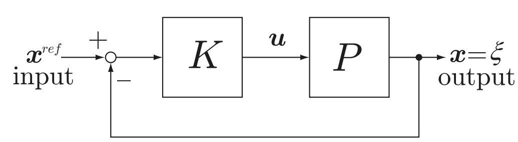

Figure 5: One DOF control system

K as;

$$K = -\frac{\partial h(\boldsymbol{x})}{\partial \boldsymbol{x}} \tag{18}$$

This equation means that to realize the autonomous system the controller K is a nonlinear function of $\boldsymbol{x}$. By substituting equation (18) into (15), the input $\boldsymbol{u}$ of the 1DOF feedback system is represented as;

$$\boldsymbol{u} = -\frac{\partial h(\boldsymbol{x})}{\partial \boldsymbol{x}}(\boldsymbol{x}^{ref} - \boldsymbol{x}) \tag{19}$$

By considering that the inputs $\boldsymbol{u}$ in equation (2) and (19) are the same, we obtain;

$$\boldsymbol{x}^{ref} = -\left(\frac{\partial h(\boldsymbol{x})}{\partial \boldsymbol{x}}\right)^{\#} h(\boldsymbol{x}) + \boldsymbol{x} + \left(\frac{\partial h(\boldsymbol{x})}{\partial \boldsymbol{x}}\right)^{\perp} \alpha \tag{20}$$

where $[\,\cdot\,]^{\perp}$ means basis of null space and $[\,\cdot\,]^{\perp}\alpha$ means an arbitrary vector that belongs to the null space. We call $\boldsymbol{x}^{ref}$ in equation (20) the pseudo-reference of the autonomous system. It is a virtual reference inside the controller, which means that an implicit reference is obtained based on the current state variable. Here we remark that $\boldsymbol{x}^{ref}$ does not always coincide with ξ because $\boldsymbol{x}^{ref}$ is obtained as a virtual reference from dynamical point of view.

3 CONTROLLER REDUCTION

3.1 Controller reduction using principal component analysis

A larger ℓ in equation (5) causes a larger number of terms in polynomial function. The number of terms L is calculated as follows;

$$L = 1 + \sum_{i=1}^{\ell} {}_nH_i = \sum_{i=0}^{\ell} \frac{(n+i-1)!}{(n-1)!i!} \tag{21}$$

where ${}_nH_i$ is a repeated combination, n and ℓ are the dimension of $\boldsymbol{x}$ and the power respectively. Table 1 shows the value of L with respect to n and ℓ. L increases rapidly according to the increase of n and ℓ. For example, in the case of the $n = 18$ which represents a planar human legged model, more than several thousand terms are required. High order dimension makes it difficult to design an autonomous system. Actually, calculation amount in polynomial function is as follows;

Table 1: The value of L with respect to n and ℓ

n \ ℓ	3	4	5	$\cdots$	10
3	20	35	56		286
4	35	70	126		1001
5	56	126	252		3003
$\vdots$					
18	1330	7315	33649		$10^7>$

$$c(n,\ell) = m\left[\sum_{i=0}^{\ell}\left\{\frac{(n+i-1)!}{(n-1)!i!}(i+1)\right\} - 1\right] \tag{22}$$

where m is the dimension of $\boldsymbol{u}$. Then by using the Stirling's approximation, the order of $c(n,\ell)$ is required more and more calculation cost as follows;

$$O^{(c)} \approx O\left(\sqrt[m]{\frac{n+\ell-1}{(n-1)\ell}}\,\frac{(n+\ell-1)^{(n+\ell-1)}}{(n-1)^{n-1}\ell^{\ell}}(\ell+1)\right) \tag{23}$$

To overcome this problem, we define a new controller formulation as;

$$\begin{aligned} \boldsymbol{u} &= a_0 + a_1\boldsymbol{x} + \widehat{a}_2\widehat{\boldsymbol{x}}^2 + \cdots + \widehat{a}_\ell\widehat{\boldsymbol{x}}^\ell & (24)\\ &= \widehat{\Theta}\widehat{\phi}(\boldsymbol{x},Q) = \widehat{h}(\boldsymbol{x},Q) & (25)\\ \widehat{\phi}(\boldsymbol{x},Q) &= \begin{bmatrix} 1 & \boldsymbol{x}^T & \widehat{\boldsymbol{x}}^{2T} & \cdots & \widehat{\boldsymbol{x}}^{\ell T}\end{bmatrix}^T & (26)\end{aligned}$$

$\widehat{\boldsymbol{x}}$ is a linear projection of $\boldsymbol{x}$ as;

$$\widehat{\boldsymbol{x}} = Q\boldsymbol{x} \quad \in R^r \tag{27}$$

where $r < n$ and $Q \in R^{r\times n}$ is a constant matrix. Q is obtained by principal component analysis of Ξ in (3) as follows;

$$\begin{aligned} \Xi &= USV^T & (28)\\ &= \begin{bmatrix} U_1 & U_2 \end{bmatrix}\begin{bmatrix} S_1 & 0 \\ 0 & S_2\end{bmatrix}\begin{bmatrix} V_1^T \\ V_2^T\end{bmatrix} & (29)\\ S_1 &= \mathrm{diag}\{\ s_1 \quad s_2 \quad \cdots \quad s_r\ \} & (30)\\ S_2 &= \mathrm{diag}\{\ s_{r+1} \quad s_{r+2} \quad \cdots \quad s_n\ \} & (31)\end{aligned}$$

By assuming $s_r \gg s_{r+1}$, Q is obtained by;

$$Q = U_1^T \tag{32}$$

In equation (26), $\boldsymbol{x}$ is hold and more than second power of $\boldsymbol{x}$ are replaced to $\widehat{\boldsymbol{x}}$ because of the conservation of observation. Projection

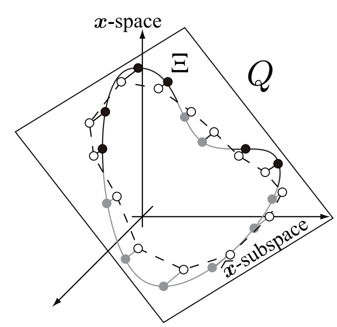

Figure 6: Projection of Ξ in $\boldsymbol{x}$-space on $\widehat{\boldsymbol{x}}$-subspace

with Q means that the most probable subspace is calculated to approximate the motion Ξ as shown in Figure 6. In the reduced controller, the value of terms $\widehat{L}$ is calculated as;

$$\widehat{L} = 1 + n + \sum_{i=2}^{\ell} {}_rH_i = 1 + n + \sum_{i=2}^{\ell}\frac{(r+i-1)!}{(r-1)!i!} \tag{33}$$

For example, in the case of $n = 18$ and $\ell = 4$, $L = 7315$ (cf. Table 1), the amount of increase of $\widehat{L}$ is smaller than that of L as shown in Table 2. Measured computational times also gradually decrease when r become smaller by using INTEL Core2Duo 2.4GHz processor for calculation. Calculation amount of reduced controller is as follows;

$$\hat{c}(n,\ell,r) = m\left[\sum_{i=2}^{\ell}\left\{\frac{(r+i-1)!}{(r-1)!i!}(i+1)\right\} + 2n - 2\right] \tag{34}$$

By using the Stirling's approximation, the order of $c(n,\ell,r)$ is calculated as follows;

$$O^{(\hat{c})} \approx O\left(\sqrt[m]{\frac{r+\ell-1}{(r-1)\ell}}\,\frac{(r+\ell-1)^{(r+\ell-1)}}{(r-1)^{r-1}\ell^{\ell}}(\ell+1)\right) \tag{35}$$

Table 2: The value of $\widehat{L}$ and computational time with respect to r

	n=18, ℓ=4							n
r	4	5	6	···	10	···	17	18
$\widehat{L}$	84	139	222	···	1009	···	5986	7315
time[ms]	2.2	4.5	8.1	···	53.2	···	469.9	596.3

When r is smaller than n, the order of $\hat{c}$ is sufficiently smaller than that of c. This result shows the effectiveness of the proposed reduction method.

It has been already shown that Θ is calculated by equation (9), and it requires inverse of $\Phi\Phi^T \in R^{L\times L}$. By the same way, $\widehat{\Theta}$ is obtained by;

$$\widehat{\Theta} = \boldsymbol{U}\widehat{\Phi}^{\#} \tag{36}$$

$$\widehat{\Phi} = \left[\begin{array}{cccc} \widehat{\phi}(\boldsymbol{x}_1, Q) & \widehat{\phi}(\boldsymbol{x}_2, Q) & \cdots & \widehat{\phi}(\boldsymbol{x}_N, Q) \end{array}\right] \tag{37}$$

and it requires the calculation of inverse of $\widehat{\Phi}\widehat{\Phi}^T \in R^{\widehat{L}\times\widehat{L}}$, which is much smaller than $\Phi\Phi^T$. Moreover, the pseudo-reference with the controller reduction requires calculation of pseudo inverse of $\partial\widehat{h}/\partial\boldsymbol{x} \in R^{\widehat{L}\times n}$ which is much smaller than that of $\partial h/\partial\boldsymbol{x} \in R^{L\times n}$ in equation (20).

3.2 Validation of controller reduction

3.2.1 Tap dancing robot

The proposed controller reduction is applied to the tap dancing robot which is a simple system for evaluation. The tap dancing robot is shown in Figure 7-(a) and its dynamical model is shown in Figure 7-(b). The detail of the dynamic equation of tap dancing robot is shown in [8]. It steps continuously by changing the stance

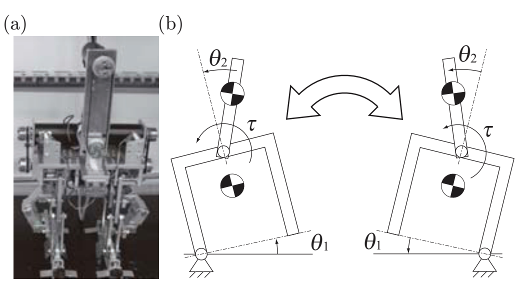

Figure 7: Tap dancing robot[8] and its dynamical model

foot and is stabilized by shaking the head. The input is the torque τ and the state variable $\boldsymbol{x}$ consists of lower body rotational angle θ_1, head rotational angle θ_2 and their angular velocities $\dot{\theta}_1$, $\dot{\theta}_2$ as follows;

$$\boldsymbol{x} = \left[\begin{array}{cc} \boldsymbol{x}_p^T & \boldsymbol{x}_v^T \end{array}\right]^T \in R^4 \tag{38}$$

$$\boldsymbol{x}_p = \left[\begin{array}{cc} \theta_1 & \theta_2 \end{array}\right]^T, \boldsymbol{x}_v = \left[\begin{array}{cc} \dot{\theta}_1 & \dot{\theta}_2 \end{array}\right]^T$$

Here we assume that the impact of foot on the ground is completely inelastic collision. The referenced motion Ξ in equation (3) is generated by giving step reference to PD controlled θ_2. The robot moves dynamically but unstably. By clipping one cycle motion, Ξ is obtained and this motion is modeled by the attractor design method.

Because $n = 4$ in this robot, L and $\widehat{L}$ are calculated for different orders of polynomial ℓ as shown in Table 3. $r = 3$ is utilized for controller reduction. $\ell = 2$ dose not stabilize the robot by both original and reduced controller (which is indicated by "failed" in Table 3). This result show that controller reduction method requires only 21 terms which is smaller value than 35 terms to stabilize the robot by using the conventional method. Therefore, $\ell = 4$ is used

Table 3: L and $\widehat{L}$ of polynomial function

The order of polynomial ℓ	2	3	4
The value of L (Original controller $n = 4$)	15 failed	35	70
The value of $\widehat{L}$ (Reduced controller $n = 4$, $r = 3$)	11 failed	21	36

to design the pseudo-reference in the following.

The pseudo-reference is designed by using the conventional controller and the reduced controller. Because $\boldsymbol{x}^{ref}$ is not decided uniquely as shown in equation (20), the objective function J_t is set as;

$$J_t = \left\|W_1(\boldsymbol{x}_p^{ref} - \boldsymbol{x}_p)\right\|^2 + \left\|W_2\boldsymbol{x}_v^{ref}\right\|^2 \tag{39}$$

where W_1 and W_2 are weighting matrices. The first term aims at making the distance small between $\boldsymbol{x}_p^{ref}$ and $\boldsymbol{x}_p$ to avoid a radical variation of $\boldsymbol{x}^{ref}$. The second term aims at making $\boldsymbol{x}_v^{ref}$ small so that the pseudo-reference shows posture information with small velocity. Figure 8 and 9 show the motion of the tap dancing motion with the conventional controller and the reduced controller respectively. (a) shows the locus of $\boldsymbol{x}$ with a solid line and the pseudo-reference with dots respectively in θ_1, $\dot{\theta}_1$ and θ_2 space. (b) shows the motion of the tap dancing robot. Indicated numbers in (b) correspond to those in (a). From these results, we can see that the

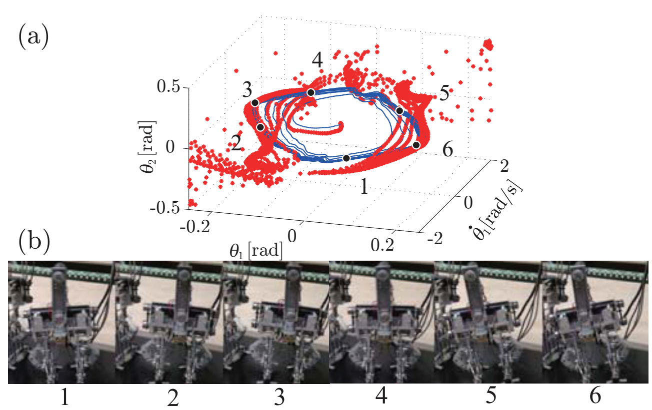

Figure 8: Tap dancing motion and pseudo-reference with original controller

stable motion is realized with the reduced controller, and pseudo-references are designed similarly with the conventional method.

4 PSEUDO-REFERENCE FOR HIGH-DEGREE OF FREEDOM SYSTEM

4.1 Human legged model

In this section, pseudo-reference is applied to human legged motion using the controller reduction. Human legged model is shown in Figure 10. The model is a high-degree of freedom model in

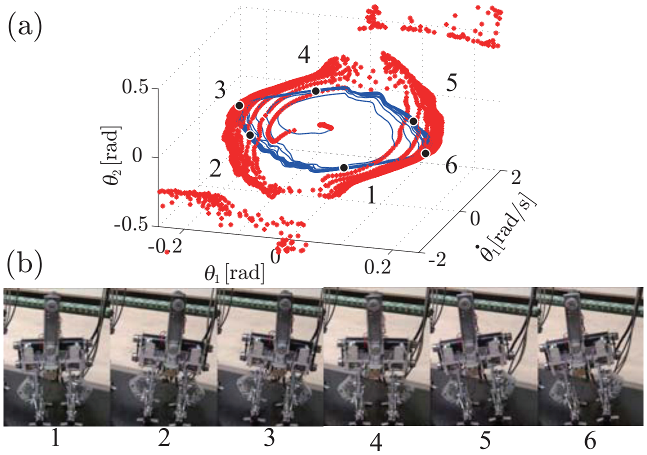

Figure 9: Tap dancing motion and pseudo-reference with controller reduction

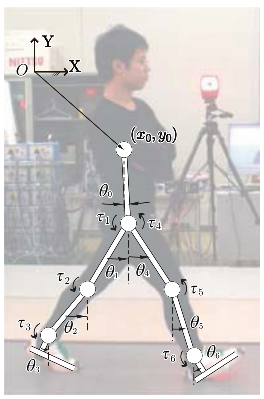

Figure 10: High-degree of freedom model for human legged motion

sagittal plane. XY coordinates are absolute coordinates, (x_0, y_0) represents the center of mass of the upper body and θ_i, τ_i $(i = 0, 1, 2, 3, 4, 5, 6)$ represent joint angles and joint torques respectively. The state variable $\boldsymbol{x}$ consists of x_0, y_0, θ_i and their velocities $\dot{x}_0$, $\dot{y}_0$, $\dot{\theta}_i$ as follows;

$$\boldsymbol{x} = \left[\begin{array}{cc} \boldsymbol{x}_p^T & \boldsymbol{x}_v^T \end{array} \right]^T \in R^{18} \quad (40)$$

$$\boldsymbol{x}_p = \left[\begin{array}{ccccc} x_0 & y_0 & \theta_0 & \cdots & \theta_6 \end{array} \right]^T \quad (41)$$

$$\boldsymbol{x}_v = \left[\begin{array}{ccccc} \dot{x}_0 & \dot{y}_0 & \dot{\theta}_0 & \cdots & \dot{\theta}_6 \end{array} \right]^T \quad (42)$$

The input $\boldsymbol{u}$ consists of each joint torque as follows;

$$\boldsymbol{u} = \left[\begin{array}{cccc} \tau_1 & \tau_2 & \cdots & \tau_6 \end{array} \right]^T \in R^6 \quad (43)$$

The controller is designed by using $\bar{\boldsymbol{x}}$ as follows;

$$\boldsymbol{u} = \widehat{\Theta}\widehat{\phi}(\boldsymbol{x}, Q) \quad (44)$$

Human walking motion is measured by an optical motion capture system and it is projected on sagittal plane to obtain Ξ in equation (3). The linear projection Q is obtained using $r = 5$. Table 4 shows relationship between the number of terms in polynomial. The original controller (without reduction) contains 7315 terms with $n = 18$, on the other hand, the reduced controller has only 139 terms by employing $r = 5$.

Table 4: Number of terms of polynomial function

Exponential number ℓ	4
Number of terms (Original controller $n = 18$)	7315
Number of terms (Reduced controller $n = 18$, $r = 5$)	139

4.2 Pseudo-reference of walking motion

Based on the reduced controller, pseudo-references of walking motion is obtained. To determine $\boldsymbol{x}^{ref}$ in equation (20) uniquely, the following objective function J_ℓ is employed and minimized.

$$\begin{aligned} J_\ell &= \left\| W_1(\boldsymbol{x}_p^{ref}[k] - \boldsymbol{x}_p[k]) \right\|^2 \\ &+ \left\| W_2(\boldsymbol{x}_p^{ref}[k+1] - (\boldsymbol{x}_p^{ref}[k] + T\boldsymbol{x}_v^{ref}[k])) \right\|^2 \end{aligned} \quad (45)$$

where W_1 and W_2 are weighting matrices and T is sampling time. The first term is same as previous section. The second term aims at satisfaction of dynamic consistency between $\boldsymbol{x}_p^{ref}$ and $\boldsymbol{x}_v^{ref}$. Because of the symmetry of walking motion, the half walking motion in the case of the right supporting leg is represented in Figure 11. The measured walking motion and pseudo-reference are represented by a solid line and a dashed line respectively. The right leg joints are marked by white circles. The posture of pseudo-reference is illustrated so that its hip joint position and orientation coincides to the measured data. From this result, pseudo-reference gives us the following information for human walking.

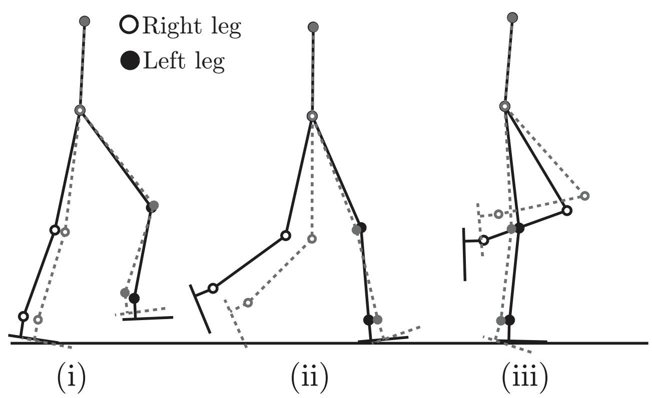

Figure 11: Human walking motion and its pseudo-reference

- In (i), the kicking leg (right leg) moves forward to slow down the body speed.
- In (ii), the swing leg is moved forward strongly for the next gait. The landing leg is stretched to support the weight of the body.
- In (iii), the swing leg is continuously lifted up to prepare for the next foot landing. The ankle of support leg is bended to obtain the propulsion force.

Here we note that

(a) Unfortunately, the obtained reduced controller could not stabilize the legged system. Because Ξ obtained from a motion capture system projecting in the sagittal plane, Ξ may not satisfy the dynamic constrain of the planner walking. Transformation of Ξ considering dynamical consistency will be required.

(b) Pseudo-reference obtained from the reduced controller should be compared to original pseudo-reference obtained from the original controller. However, the original controller is not calculated because of the shortage of computational memory because of large number of L.

5 Conclusions

In this paper, we proposed the controller reduction method to design an autonomous controlled system, and pseudo-reference is applied to human motion based on the proposed reduction method. The results of this paper are summarized as follows;

1. Controller reduction of an autonomous control system based on an orbit attractor is proposed by using correlation of motion data.

2. The proposed method is evaluated by the tap dancing robot. The same tap dancing motion and pseudo-reference are realized by using either the proposed controller or the conventional one, which means the effectiveness of the proposed reduction method.

3. The proposed method is also applied to human legged motion which is multi-degree of freedom system. The pseudo-reference showed the significant information to realize the walking motion.

The pseudo-references are designed as movie of postures which contain significant information. In the future, motion instruction for beginner will be performed by using the movie of expert's pseudo-references.

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