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Attractor Design of Nonlinear Dynamics based on Energy Distance in State Space

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Abstract

Robot motions are generated based on stabilizing controllers and reference motion patterns. On the other hand, human motions are determined through the interaction between the body and its environments. Motion patterns are not prepared a priori but generated as the results of the entrainment phenomena of the dynamics. So far, we have developed a controller design method that makes a dynamics entrain to the specific closed curved line. However, the obtained attractor is sometimes different from a desired one. That is fatal for a robot motion with a drastic change of the dynamic equation of the robot body through the motion. In this paper, we develop a new attractor design method based on energy distance in the state space.

keywords : *Attractor design, Nonlinear dynamics, Dynamics-based information processing, Hamiltonian, Motion emergence*

1. Introduction

For industrial robots, the robot control systems have been designed using refrence motion patterns and stabilizing controllers as shown in figure 1. The

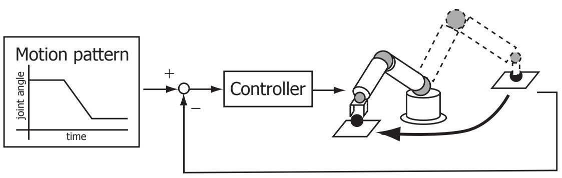

Fig.1 Motion control system for industrial robots

reference patterns are designed considering the environments where the robot works and the controllers are designed based on the robot body dynamics. For the industrial robot, because the environments are fixed and the purpose of the control system is a precise task execution, the conventional control system design methods are suitable. The motion patterns are designed so that it satisfies the dynamical constrains of the robot body and environments, the stability of the closed loop system is entrusted to the robustness of the controller. The same strategies are utilized for humanoid robots. However, because humanoid robots move in unknown environments, the fixed motion patterns are not appropriate and the robustness of the controller is not sufficient. Another control method that defines the motion pattern autonomously in real-time is required in the changing environments.

On the other hands, the human motions are generated through the interaction between body dynamics, information processing and environments as shown in figure 2. The motion patterns are not prepared a priori but emerged as the results of the interaction. This concept corresponds with "embodiment"[1] that represents a close relationship between body and intelligence in the brain science research field.

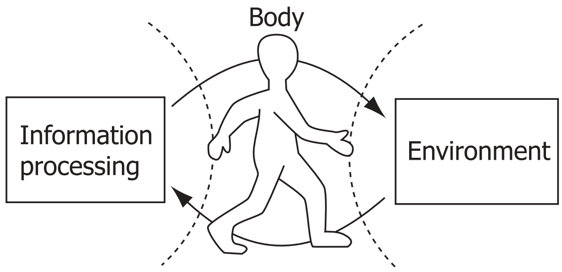

Fig.2 Motion generation of the human

From the robotics and control engineering points of view, the robot motion generated through the interaction between the body and environments, interpreted as the entrainment phenomenon of the nonlinear dynamics, and the generated motion pattern corresponds to an attractor. The robot body dynamics is controlled by an information processing system so that it entrains to a closed curved line from any initial positions, which yields the motion and motion pattern. Based on this concept, we have proposed an attractor design method that leads the motion emergence of the humanoid robot [2]. In this method, we set a desirable motion and design a controller with a polynomial representation in the state space of the robot dynamics based on the dynamics-based information processing system theory [3]. The designed controller does not have physical meanings in itself but generates an attractor by making a closed loop system with the robot body dynamics. In reference [3], the bending knee motion of the humanoid robot is realized. Though the controller is calculated using a least square method that minimizes the input power using multi-step ahead prediction of the state variable, a contradiction is caused in causality between input and trajectory generation. That yields a large difference between the desired trajectory and generated motion, which is a fatal error when the robot body dynamics is drastically changed while its motion. In this paper, the problem of the conventional method is declared and modified methods are proposed based on the energy distance of the state space.

2. Attractor Design Method

2·1 Minimization of the input power

In this section, I will illustrate the attractor design method in reference [3]. For simplicity, the dynamics is assumed to be controllable and represented by the following linear discrete time difference equation,

$$x[k+1] = Ax[k] + Bu[k] \tag{1}$$

where $x[k] \in \boldsymbol{R}^n$ is a state variable and $u[k] \in \boldsymbol{R}^m$ is an input signal. In the design algorithm, $u[k]$ is represented by the function of $x[k]$ such that

$$u[k] = f(x[k]) \tag{2}$$

and the closed loop system

$$x[k+1] = Ax[k] + Bf(x[k]) \tag{3}$$

has an attractor on the desirable closed curved line Ξ

$$\Xi = \begin{bmatrix} \xi_1 & \xi_2 & \cdots & \xi_N \end{bmatrix} \tag{4}$$

The controller is designed by defining a vector field so that Ξ becomes an attractor, setting the pairs of ($u[k]$, $x[k]$) and approximating $f(x[k])$ by ℓ-th order polynomial of $x[k]$.

The parameters A and B in equation (1) and Ξ are assumed to be given. And it is assumed that Ξ is realizable, which means the sequence of input signal $u[k](k = 1, 2, \cdots)$ that moves the state variable along with Ξ exists. The controller is designed as follows. First, set x_i in x-space and find ξ_i that is nearest to x_i. x_{i+j} is j-step ahead prediction of x_i that is represented by the following equation.

$$x_{i+j} = A^j x_i + \Gamma U \tag{5}$$

$$\Gamma = \begin{bmatrix} B & AB & \cdots & A^{j-1}B \end{bmatrix} \tag{6}$$

$$U = \begin{bmatrix} u_i^T & u_{i+1}^T & \cdots & u_{i+j-1}^T \end{bmatrix}^T \tag{7}$$

Because Γ is an extended controllable matrix with column full rank when $j \geq n$, U that takes x_{i+j} onto ξ_{i+j} exists and obtained by

$$U = \Gamma^{\#} \left(\xi_{i+j} - A^j x_i \right) \tag{8}$$

By using the obtained U, we calculate x_k, $(k = i, \cdots, i+j-1)$ and obtain pairs of (x, u). By defining many initial points x_i and obtain many pairs of (x, u), $f(x[k])$ in equation (2) is obtained by polynomial approximation.

2·2 Contradiction in causality

Though x_i is guaranteed to coincide to ξ_{i+j} in j-step ahead, the route is not specified a priori. Equation (8) means the minimization of the input power, and x_{i+1}, x_{i+2}, $\cdots$, x_{i+j-1} is defined subsequently, which is not guaranteed to pass near Ξ. As shown in figure 3, when

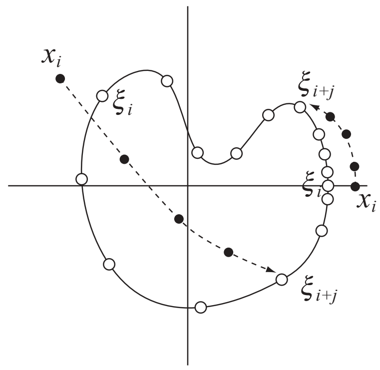

Fig.3 Desired closed curved line and trajectory

the motion of the dynamics is slow, the conventional method is effective, however in contrast, the obtained trajectory makes a short cut and can be distant from Ξ. The following results show an example.

Consider the inverted pendulum system as shown in figure 4. Setting θ (the rotational angle of the pen-

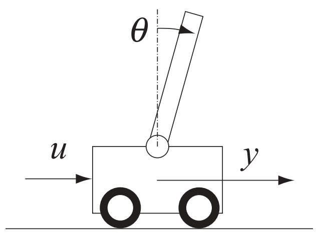

Fig.4 Inverted pendulum system

dulum), y(position of the cart), u (input force) and defining the state vector x as follows,

$$x = \begin{bmatrix} \theta & \dot{\theta} & y & \dot{y} \end{bmatrix}^T \tag{9}$$

the dynamic equation is obtained. By discretizing the dynamics in a sampling time T, we obtain a discrete time dynamics in equation (1). By setting Ξ, the attractor is designed. Figure 5 shows Ξ and calculated x_k, $(k = i, \cdots, i+30)$. Though x is a 4 dimensional

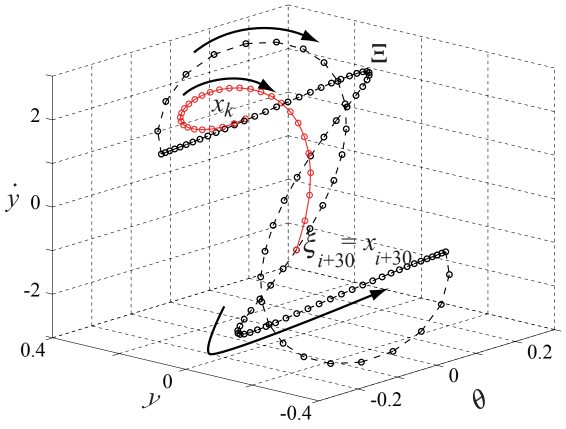

Fig.5 Trajectory of obtained x via conventional method

vector, 3 dimensional space whose coordinates are θ, y and $\dot{y}$ are shown in figure 5. As shown in this figure, the obtained x_i, x_{i+1}, $\cdots$ are not along with the desired trajectory. Figure 6 shows $\|\xi_k - x_k\|$ using obtained x_k. $\|\xi_k - x_k\|$ does not decrease, which means

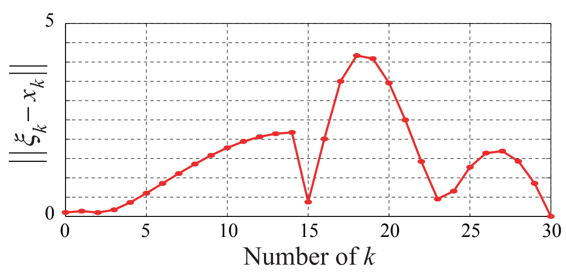

Fig.6 Norm of $\|\xi_k - x_k\|$ via conventional method

x_k does not go along with ξ_k. From the obtained pairs of (x, u), $f(x[k])$ is approximated by 6th-order polynomial. Figure 7 shows the trajectory of the controlled dynamics starting from two initial points represented by $*$. Though the dynamics is stabilized to one attractor, it is different from Ξ. In this example, because the control object is a linear system, the difference between the obtained trajectory and Ξ is not fatal error. When the system is nonlinear, we use the linear approximated system A_k, B_k around each ξ_k in the attractor designed stage. Γ is set assuming x_k goes along ξ_k, which means the difference between the obtained trajectory and Ξ is fatal. This problem is led by the contradiction in causality related to the input signal and trajectory, which causes unstable motion when the dynamic characteristic drastically changes through the motion such as walking. In the following section, I propose a modified attractor design method.

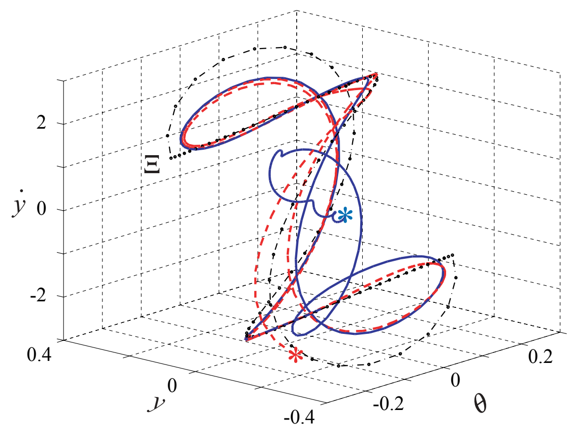

Fig.7 Trajectory of the dynamics

3. Trajectory-based Attractor Design Method

3·1 Euclid distance-based design method

In this section, I show an attractor design method minimizing the distance between x_k and ξ_k. From equation (1), we obtain the following equations.

$$X_{k+1} = \boldsymbol{A}x[k] + \boldsymbol{B}U \tag{10}$$

$$X_{k+1} = \begin{bmatrix} x[k+1] \\ x[k+2] \\ \vdots \\ x[k+j] \end{bmatrix}, \ \boldsymbol{A} = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^j \end{bmatrix} \tag{11}$$

$$\boldsymbol{B} = \begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{j-1}B & A^{j-2}B & \cdots & B \end{bmatrix} \tag{12}$$

Based on these equations, we obtain U as follows.

$$U = \boldsymbol{B}^{\#} \left(\Xi_{k+1} - \boldsymbol{A}x_i \right) \tag{13}$$

$$\Xi_{k+1} = \begin{bmatrix} \xi_{i+1}^T & \xi_{i+2}^T & \cdots & \xi_{i+j}^T \end{bmatrix}^T \tag{14}$$

This method means the minimization of the following criterion function J,

$$J = \sum_{\kappa=1}^{j} \|\xi_{i+\kappa} - x_{i+\kappa}\| \tag{15}$$

which corresponds to the square summation of euclid distance between x_k and ξ_k as sown in figure 8. We design an attractor using equation (13). Figure 9 shows Ξ and one example of x_k, $(k = i, \cdots, i+30)$ like figure 5. Comparing figure 5, x_k goes along ξ_k. Figure 10 shows the obtaind euclid norm $\|\xi_k - x_k\|$. x_k approaches ξ_k with increasing k. Figure 11 shows the motion of the controlled dynamics. The controller is

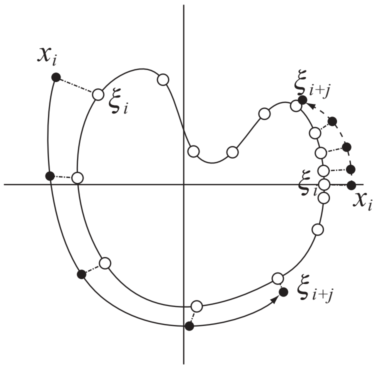

Fig.8 Definition of Euclid distance in the criterion function

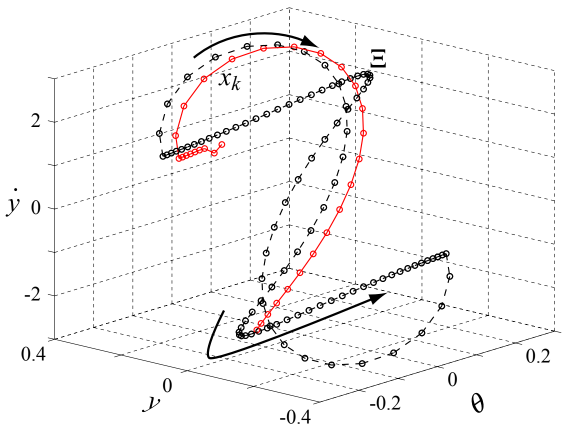

Fig.9 Trajectory of obtained x via the least square method

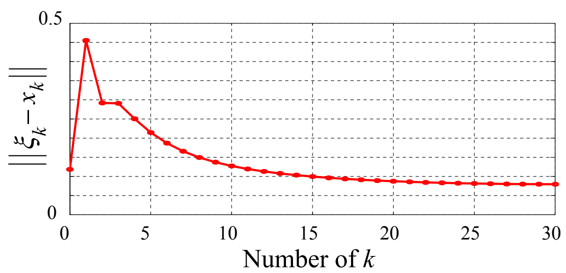

Fig.10 Norm of $\|\xi_k - x_k\|$ via the least square method

designed using 6-th order polynomial same as previous method. Comparing figure 7, the obtained trajectory is similar to Ξ.

3·2 Energy distance-based design method

In the previous section, the dynamics does not converge to the attractor as shown in figure 11. $x[k]$ takes other routes in each cycle and the trajectory draws thick line in figure 11. This is because the convergence rate of $\|\xi_k - x_k\|$ in figure 10 is low. In the following, I consider the modifying method.

When a state variable $x[k]$ of a stable dynamics con-

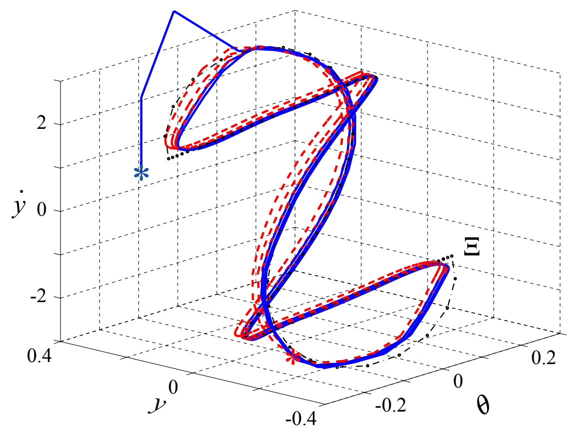

Fig.11 Trajectory of the dynamics via the least square method

verges to zero, euclid distance $\|x[k]\|$ does not correspond to how the state vector $x[k]$ approaches to zero. Figure 12 shows a concept chart. The upper figure

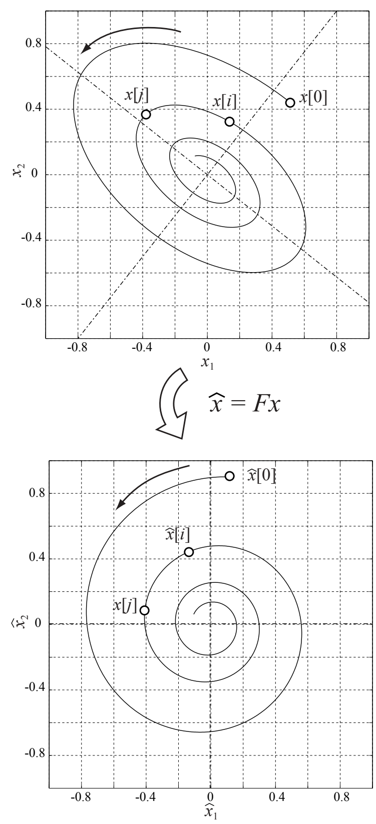

Fig.12 Convergence of dynamics

shows the trajectory of $x[k]$ starting from $x[0]$ in state space. $\|x[j]\| < \|x[i]\|$ $(j < i)$ is not always satisfied, which means $x[j]$ is more convergent than $x[i]$ but $\|x[j]\|$ can be likely larger than $\|x[i]\|$. On the other hand, as shown in the lower figure, x-coordinates is transformed to $\hat{x}$-coordinates by the transform matrix F and when $\|\hat{x}[j]\| < \|\hat{x}[i]\|$ $(j < i)$ is always satisfied, euclid distance $\|\widehat{x}\|$ corresponds to the convergence rate of the state variable. In the following, calculation method of F is explained.

Consider a conservative system represented by the following differential equation.

$$\dot{x} = Ax, \quad x \in R^n \tag{16}$$

where we assume that A is diagonalizable. The state variable x starting from x_0 moves on the shell of a ellipsoid in n-dimensional space with its center on the origin. Let's obtain a matrix F that transforms the ellipsoid to sphere. Consider eigen value decomposition of A as follows.

$$A = T\Lambda T^{-1} \tag{17}$$

$$\Lambda = \mathrm{diag}\left\{ \lambda_1, \quad \lambda_2, \quad \cdots, \quad \lambda_n \right\} \tag{18}$$

Because the dynamics is a conservative system,

$$\mathrm{Real}(\lambda(A)) = 0 \tag{19}$$

is satisfied. Here, we consider $\widetilde{x}$ defined by the following transformation.

$$\widetilde{x} = T^{-1}x \tag{20}$$

$\widetilde{x}(t)$ is represented by the following equation

$$\widetilde{x}(t) = \exp(\Lambda t)\widetilde{x}_0 = \mathrm{diag}\left\{ e^{\lambda_1 t}, \quad \cdots, \quad e^{\lambda_n t} \right\}\widetilde{x}_0 \tag{21}$$

where $\widetilde{x}_0$ means the initial value. The inner product of $\widetilde{x}(t)$ satisfies the following equation.

$$\widetilde{x}^*(t)\widetilde{x}(t) = x^T(t)\left(T^{-1}\right)^* T^{-1}x(t) \tag{22}$$

$$= \widetilde{x}_0^*\mathrm{diag}\left\{ e^{(\lambda_1^*+\lambda_1)t}, \quad \cdots, \quad e^{(\lambda_n^*+\lambda_n)t} \right\}\widetilde{x}_0 \tag{23}$$

$$= \widetilde{x}_0^*\widetilde{x}_0 = \mathrm{Const.} \tag{24}$$

Because this equation represents a sphere on $\widetilde{x}(t)$ and an ellipsoid on $x(t)$, the singular value decomposition

$$\left(T^{-1}\right)^* T^{-1} = USU^T \tag{25}$$

gives the transformation matrix F

$$\widehat{x} = Fx \tag{26}$$

$$F = S^{\frac{1}{2}}U^T \tag{27}$$

that transforms the ellipsoid in x-space to a sphere in $\widehat{x}$-space. Equation (24) means Hamiltonian (conservation value) that corresponds to the energy of the system. In the case of dissipation system, because the equation (26) represents a energy of $\|\widehat{x}(t)\|$, euclid distance of $\widehat{x}(t)$ represents the energy distance of $x(t)$.

From these considerations, the attractor design method is modified as follows using F.

1. Using defined x_i, find ξ_i that minimizes $\|F(\xi_i - x_i)\|$.
2. Define j in equation (10) that satisfies $\|F(\xi_{i+j} - x_{i+j})\| < \Delta$, where Δ is a design parameter. j is calculated by

$$j = \frac{\log\Delta - \log(\|F(\xi_i - x_i)\|)}{\log\delta} \tag{28}$$

$$\delta^j\|F(\xi_i - x_i)\| < \Delta \tag{29}$$

   where δ defines the convergence velocity of $x(t)$.
3. By using a weighted least square method based of the following evaluation function,

$$J = \sum_{k=1}^{j}\delta^{-k}\left\|F\left(\xi_{i+k} - x_{i+k}\right)\right\| \tag{30}$$

   substituting for equation (15), energy distance is evaluated.

Using the modified design method, we design a controller for the inverted pendulum system. Figure 13 shows the obtained x_k, $(k = i, \cdots, i+24)$, and figure 14 shows the value of $\|F(\xi_k - x_k)\|$. x_k converges to

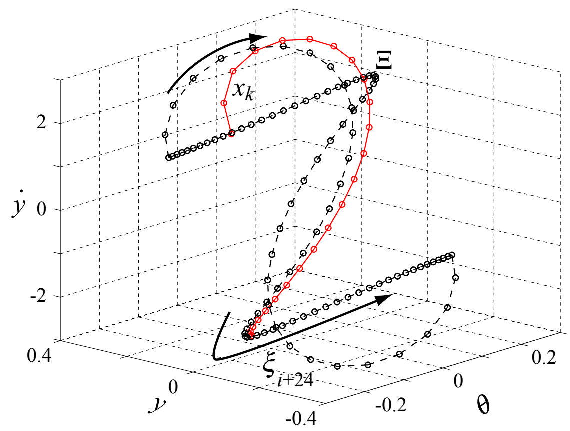

Fig.13 Trajectory of obtained x via energy distance method

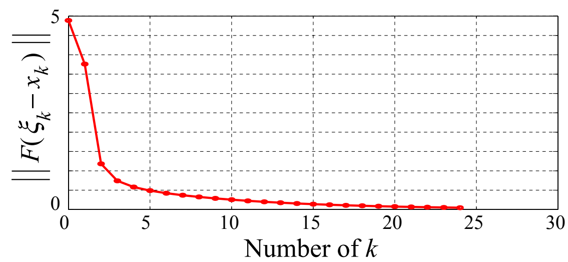

Fig.14 Norm of $\|F(\xi_k - x_k)\|$ via energy distance method

ξ_k in the sense of energy distance while k increases. Figure 15 shows the designed attractor. Comparing to figure 11, $x[k]$ approaches Ξ and one trajectory is emerged.

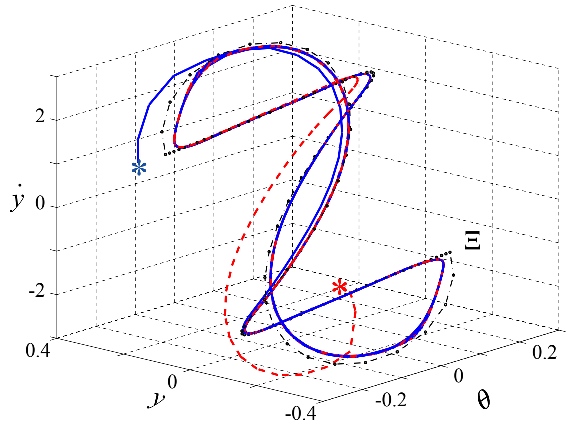

Fig.15 Trajectory of the dynamics via energy distance method

4. Conclusions

In this paper, I proposed the modified attractor design method from the linear control engineering point of view.

1. I declare the problem of the conventional method (causality of relation between the route and the input) that is the minimization of the input power.
2. To overcome the problem, I propose the modified method that minimizes euclid distance of multi-steps ahead and the desired trajectory.
3. Moreover, we propose a new approach that evaluates the energy distance of the state variable, and shows that the designed attractor approaches the desired trajectory.

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References

[1] R.Pfeifer and C.Scheier : Understanding Intelligence, Bradford Books, 2001.

[2] M. Okada, K. Osato and Y. Nakamura : Motion Emergence of Humanoid Robots by an Attractor Design of a Nonlinear Dynamics, Proc. of the IEEE International Conference on Robotics and Automation (ICRA'05), pp.18-23, 2005

[3] M. Okada, K. Tatani and Y. Nakamura Polynomial Design of the Nonlinear Dynamics for the Brain-Like Information Processing of Whole Body Motion, Proc. of IEEE International Conference on Robotics and Automation, pp.1410-1415, 2002