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Amenity Design for Congestion Reduction based on Continuum Model of Swarm

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Abstract

Control of swarm is effective for cooperative work, congestion reduction and evacuation guidance. In this paper, we propose a modeling and control method of swarm based on flow field for amenity design of our environments. The swarm is modeled by continuum fluid and its behavior is represented by flow field with an attractor. The congestion of swarm is analyzed based on the fluid density which is obtained by a steady solution of the continuity equation of compressive fluid. Based on the obtained density, a space design method is proposed to reduce the congestion using static elements that changes the flow field. The effectiveness of the proposed modeling and space design method are evaluated by simulations.

Key Words swarm control, congestion reduction, continuum model of swarm, continuity equation

1. Introduction

For our safe and comfortable life, amenity space plays an important role, which is a significant issue for urban city planning with increasing population. For example, there are so many overcrowded places in our environments such as a station premise, stairs and doorways. In these places, we feel discomfort or some accidents will be caused. To overcome these problems, the amenity space design for congestion reduction is required.

As shown in Fig.1, the casus of congestion are both behavior and number (or density) of individuals, and to reduce the congestion, behavior analysis and control of individuals are necessary. For swarm robot control, the respective control for individuals has been employed. Hirata [2] and Fink [1] proposed an explicit command control method for cooperative work of swarm. Kurabayashi [3] proposed an implicit command method for formation control of multi-robots system using nonlinear oscillator. However, these methods intend swarm with small number of individuals, and the respective control of individuals costs so much calculations and requires a complicated algorithm for great amount of individuals.

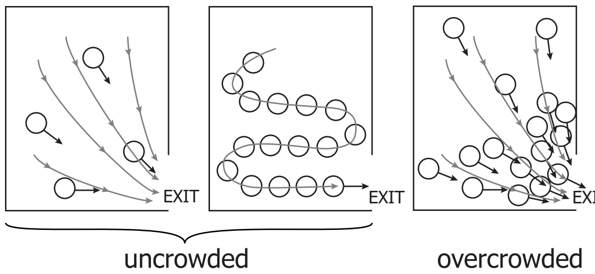

Fig.1 Swarm behavior and congestion

In this paper, we propose a continuum model of swarm. The swarm is modeled by compressive fluid and its behavior is modeled by flow field in the space. The behavior of swarm is analyzed based on continuity equation that is a hydroscopic momentum conservation law, and the congestion is calculated by its density. Moreover, based on the proposed model, the amenity space design method is proposed that realizes congestion reduction. Some modeling methods of swarm based on a hydroscopic dynamics have been proposed [5]. They use Navier-Stokes equation that is a hydroscopic dynamics, and incompressibility of fluid is assumed for simplicity. On the other hand, in this paper, because the congestion analysis is a target, compressive fluid is considered and continuity equation which is simpler than Navier-Stokes equation is used. The effectiveness of our modeling and amenity space design method are evaluated by simulations.

2. Continuum model of swarm

2.1 Environmental formulation

In this section, the target environment is set. Consider the trajectory of human path as shown in Fig.2 for example. The trajectory of human motion in an

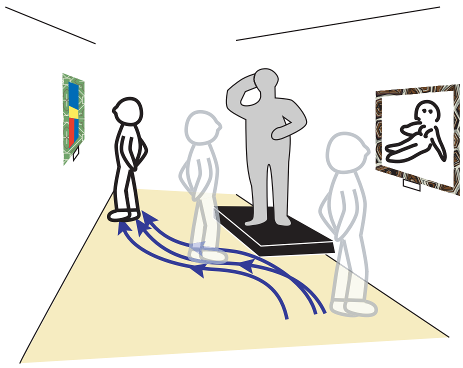

Fig.2 Human trajectory in an art museum

art museum is represented by flow field and most of people path through this trajectory. Here we set the assumptions of the environments as:

A1 The environment is two dimensional closed space enclosed by a wall without comings and goings of agents.

A2 The trajectory is cyclic.

2.2 Modeling of the behavior of swarm

The behavior of the swarm is represented by flow field in the closed space as shown in Fig.3. The flow

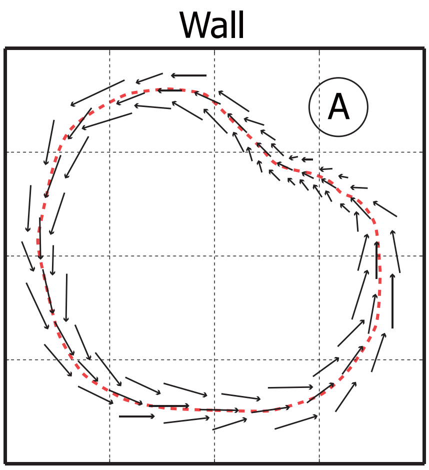

Fig.3 Configuration of trajectory in the closed space

is represented by the arrows and the length of arrows means the velocity of individual at the corresponding points. For example, because there is an attractive display at A in Fig.3, the velocity in front of A is small. The typical trajectory of individuals is represented by the red dashed line. All individuals are assumed to move according to this flow field without collision of each other.

2.3 Calculation of swarm congestion

Based on the given flow field $\boldsymbol{f}$

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} f_x(\boldsymbol{x}) \\ f_y(\boldsymbol{x}) \end{bmatrix} \tag{1}$$

at point $\boldsymbol{x} = \begin{bmatrix} x, & y \end{bmatrix}^T$ in the space, the congestion of swarm is calculated by the density of fluid using continuity equation of compressive fluid:

$$\frac{\partial \rho}{\partial t} + \rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) + v_x\frac{\partial \rho}{\partial x} + v_y\frac{\partial \rho}{\partial y} = 0 \tag{2}$$

where $\rho = \rho(\boldsymbol{x}, t)$ is the density, $v_x = v_x(\boldsymbol{x}, t)$ and $v_y = v_y(\boldsymbol{x}, t)$ mean the velocity of fluid at point $\boldsymbol{x}$ and time t. This equation means the time variation of density ρ at one unit square is obtained by the difference of the mass that flows out and into as shown in Fig.4. The velocity v_x and v_y are represented by

$$v_x(\boldsymbol{x}, t) = f_x(\boldsymbol{x}) - \kappa\frac{\partial \rho(\boldsymbol{x}, t)}{\partial x} \tag{3}$$

$$v_y(\boldsymbol{x}, t) = f_y(\boldsymbol{x}) - \kappa\frac{\partial \rho(\boldsymbol{x}, t)}{\partial y} \tag{4}$$

The second terms of the right hand side in equations

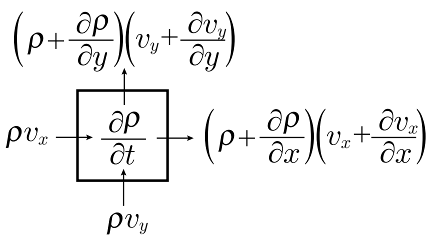

Fig.4 Continuity equation

(3) and (4) represent the diffusion terms of fluid that corresponds to the repulsive terms of individuals to avoid collision. κ is constant.

3. Behavioral simulation of swarm

3.1 Set of the flow field

Based on continuity equation, the swarm behavior is simulated. The flow field is set by the following steps.

Typical trajectory In the closed space, the typical trajectory is set as shown in Fig.5. The length of the arrow shows the velocity of individuals.

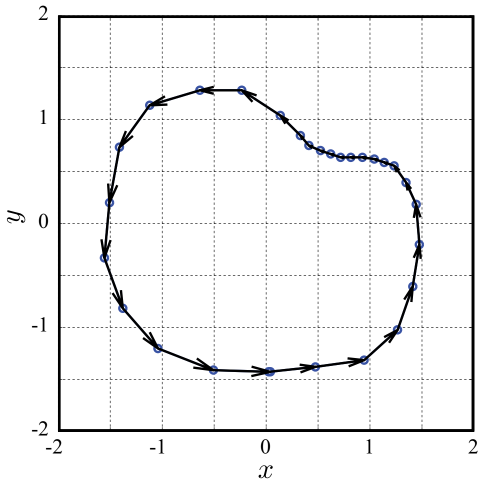

Fig.5 Typical trajectory

Flow field Based on the attractor design method in [4], flow field is designed so that the typical trajectory becomes an attractor of swarm because each individual path through without straying from the trajectory. Set a point $\boldsymbol{x}_i$ around the point of trajectory and draw a vector $\boldsymbol{f}(\boldsymbol{x}_i)$ toward the next point as shown in Fig.6. By defining many points $\boldsymbol{x}_i$, the

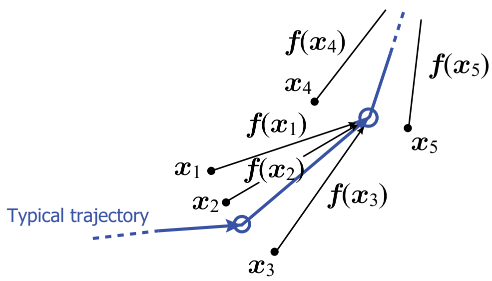

Fig.6 Flow field definition

pairs of $(\boldsymbol{x}_i,\ \boldsymbol{f}(\boldsymbol{x}_i))$ are obtained. By functional approximation, we obtain flow field $\boldsymbol{f}(\boldsymbol{x})$. According to [4], $\boldsymbol{f}(\boldsymbol{x})$ is approximated by the polynomial of ℓ-th order power-of-$\boldsymbol{x}$ as

$$\boldsymbol{f}(\boldsymbol{x}) = \Theta\phi(\boldsymbol{x}) \tag{5}$$

where Θ is a coefficient matrix of polynomial, $\phi(\boldsymbol{x})$ represents the power vector of $\boldsymbol{x}$ defined by

$$\phi(\boldsymbol{x}) = \begin{bmatrix} 1 & x & y & x^2 & xy & y^2 & \cdots & y^\ell \end{bmatrix}^T \tag{6}$$

Θ is obtained by the least square method as

$$\Theta = F\Phi^{\#} \tag{7}$$

$$F = \begin{bmatrix} \boldsymbol{f}(\boldsymbol{x}_1) & \boldsymbol{f}(\boldsymbol{x}_2) & \cdots & \boldsymbol{f}(\boldsymbol{x}_n) \end{bmatrix} \tag{8}$$

$$\Phi = \begin{bmatrix} \phi(\boldsymbol{x}_1) & \phi(\boldsymbol{x}_2) & \cdots & \phi(\boldsymbol{x}_n) \end{bmatrix} \tag{9}$$

The obtained flow field is shown in Fig.7. By functional approximation, a continuous flow field is obtained.

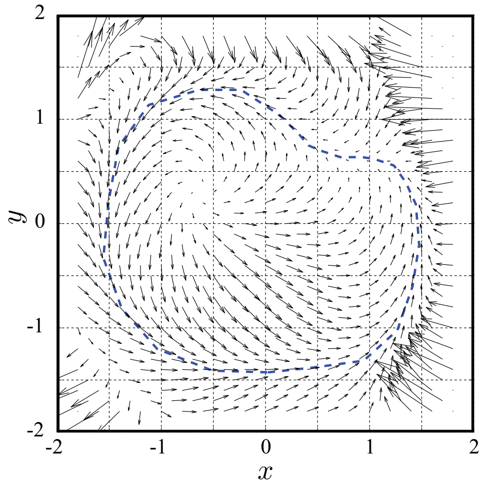

Fig.7 Flow field obtained by the functional approximation

3.2 Calculations of density

Based on the obtained flow field, the stationary solution of continuity equation is calculated using the finite elements method (FEM) with 51×51 elements. The result is shown in Fig.8. The maximum value of density is 4.0×10^{-3} at $\boldsymbol{x} = (1.12,\ 0.56)$.

3.3 Comparison with the individual motion calculations

For comparison, the motion of the swarm with 100 individuals is calculated. The stationary state is shown in Fig.9. The congestion is arose at B where is indicated by the circle. Based on this result, the time average number of individuals is calculated which is shown in Fig.10. Because the number of individuals is finite, the accurate value of density in continuous space can not be calculated, and the average of nine neighborhoods of elements is approximated. The maximum value is 0.38 at $\boldsymbol{x} = (1.36,\ 0.48)$. The trend of the result is similar to Fig.8, which shows the effectiveness of the proposed continuum model of swarm and its behavior.

4. Amenity design for congestion reduction

4.1 Change of flow field

To reduce the congestion, it is necessary to change flow field. It is not realistic in the real environment to increase the flow velocity in the area where the congestion is arisen. In this paper, set of some poles

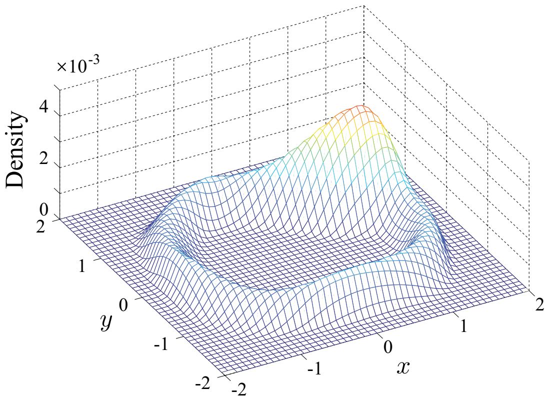

Fig.8 Density of swarm calculated from continuity equation

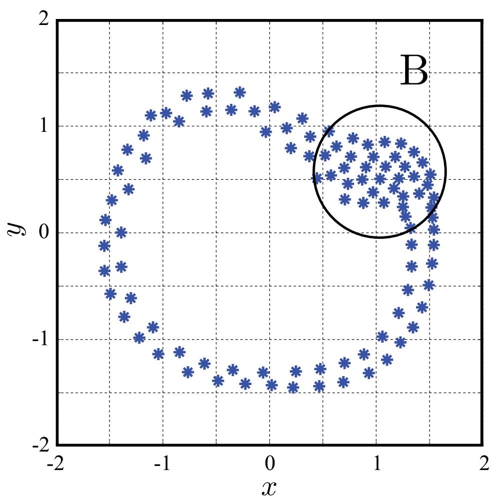

Fig.9 Motion of the individuals in swarm

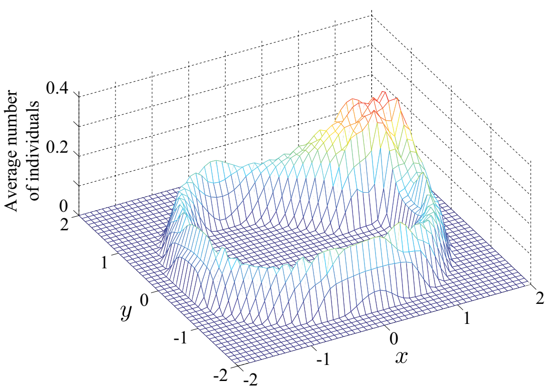

Fig.10 Average number of individuals of swarm calculated from motion

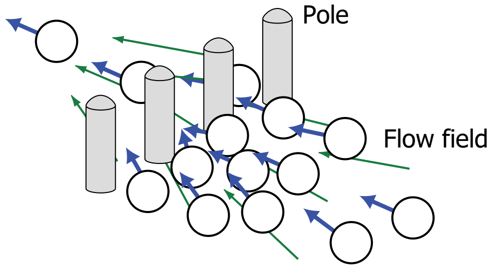

Fig.11 Pole set that stems the flow

that stem flow of individuals is employed as sown in Fig.11. The flow field around a pole is defined by

$$\boldsymbol{v}_p = \beta \frac{\boldsymbol{r}}{(\|\boldsymbol{r}\| + \alpha)^2} \tag{10}$$

with constant α and β, which is shown in Fig.12. $\boldsymbol{r}$ is a distance from the pole. This flow field means that

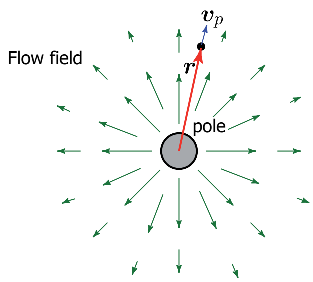

Fig.12 Flow field around the pole

each individual avoids the collision with a pole.

4.2 Effects of pole and its positioning policy

The number of individuals or summation of density in the space is constant because the space is closed. To reduce the maximum value of density, the change of density distribution is required. To position the poles affects the density of swarm as shown in Fig.13. In the back of the pole, the density is in-

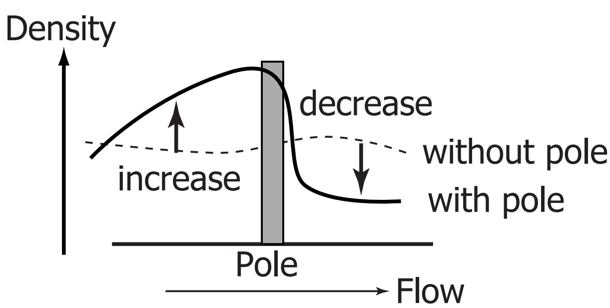

Fig.13 Effect of pole for density

creased and in the front of the pole, the density is decreased. By considering this effect, the positioning of the pole is decided as the following policy.

1. From Fig.8, the ridge line of density is obtained as shown in Fig.14.

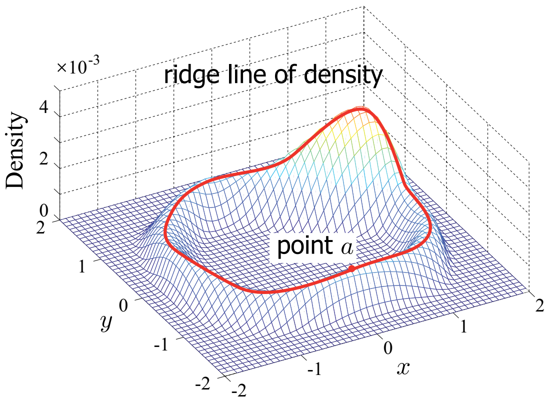

Fig.14 Ridge line of density

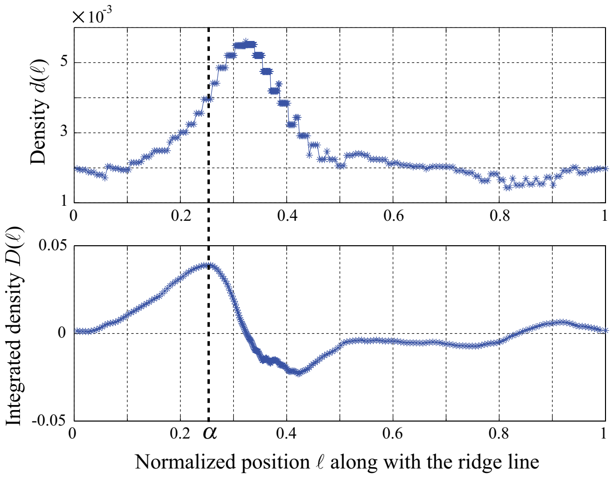

Fig.15 Density and integrated density along with the ridge line

2. Density $d(\ell)$ and integrated density $D(\ell)$ defined by

$$D(\ell) = \int_{\ell}^{\ell+\ell_0} d(s)\ ds - \int_{\ell-\ell_0}^{\ell} d(s)\ ds \qquad (11)$$

is calculated as shown in Fig.15, where ℓ is normalized position along with the ridge line starting from point a and ℓ_0 is constant. By considering the effect of the pole, we set the candidate of the pole positioning where $D(\ell)$ is maximized, which is represented by α in Fig.15.

3. We set the three poles set that prevent the flow of individuals as shown in Fig.16. Three poles are set being orthogonal to the vector field at α.

4.3 Calculations of density in the amenity space

By using the poles, the modified density is calculated based on continuity equation. The stationary

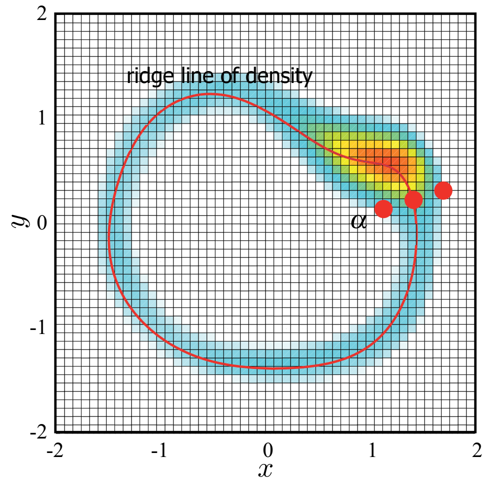

Fig.16 Positioning of three poles set

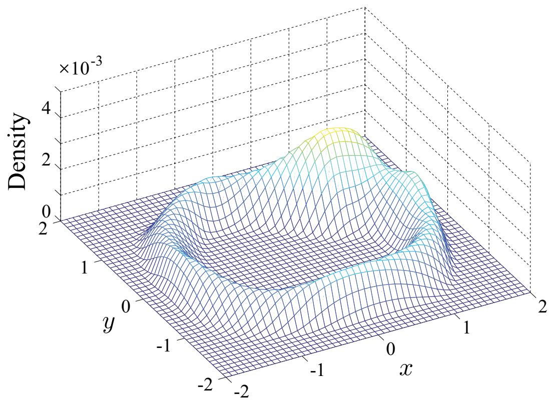

Fig.17 Density distribution of swarm with poles

solution is shown in Fig.17. The maximum value of density is 3.2×10^{-3} at $\boldsymbol{x} = (0.8,\ 0.72)$, which is 20% decrease comparing to the original one. Same as the previous results, the stationary state of the motion of individuals in the modified space is shown in Fig.18 and the time average number of the individuals is shown in Fig.19. The maximum value of the approximated density is 0.34 at $\boldsymbol{x} = (1.04,\ 0.72)$. Because the congestion is reduced, we can conclude that an amenity space is designed.

5. Conclusions

In this paper, we focus on the amenity design for congestion reduction. The results of this paper are as follows:

- The modeling method of swarm by continuum and its behavior by fluid field are proposed.
- Based on continuity equation of compressive fluid, the congestion of swarm is calculated as a density of fluid.
- Based on the obtained density distribution, an amenity space design method using poles that

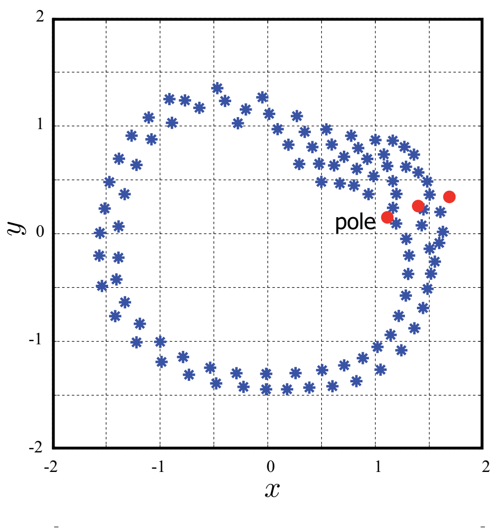

Fig.18 Motion of the individuals in swarm with pole set

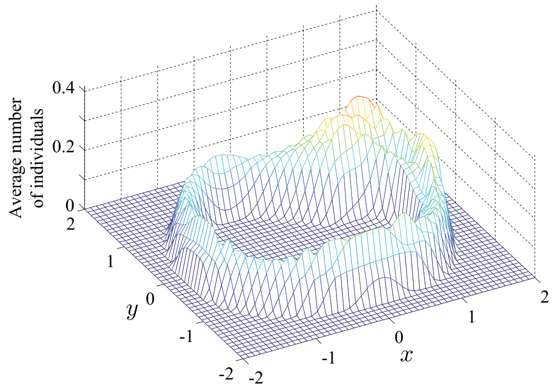

Fig.19 Average number of the individuals with pole set

stems the flow, is proposed.

- The effectiveness of modeling, congestion calculations and amenity design method are evaluated by the simulations.

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