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Polynomial Design of the Nonlinear Dynamics for the Brain-Like Information Processing of Whole Body Motion

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Abstract

For the development of the intelligent robot with many degree-of-freedom, the reduction of the whole body motion and the implementation of the brain-like information system is necessary. In this paper, we propose the reduction method of the whole body motion based on the singular value decomposition and design method of the brain-like information processing system using the nonlinear dynamics network with the polynomial configuration. By using the proposed method, we design the humanoid whole body motion that is caused by the input sensor signals.

Keywords: Brain-like information processing, Whole body motion, Reduction method, Polynomial design of the nonlinear dynamics

1 Introduction

The robot intelligence has been discussed in many researches. The brain-like information processing system is expected to be a new approach as the adaptable system on behalf of the pattern matching methods. The focus of the recent researches is the system design strategy to realize the memorization and association with the interaction between robots and their environment. In this paper, we develop the new approach to deal with the humanoid whole body motion using nonlinear dynamics.

Because the whole body motion of the humanoid robots consists of the many joint angle data, it requires much computation quantity to deal with some motions. The reduction and the symbolization of the whole body motion are necessary. In this paper, we propose a reduction and symbolization method of the whole body motion based on the principal component analysis[11] using singular value decomposition.

On the other hand, since Freeman showed the dynamical phenomenon in organisms such as entrainment and chaos, which exist in the rabbit olfactory[1, 2, 3], some researchers have tried to realize the dynamics based brain-like information processing system. These systems have been mainly developed using a neural network configuration. However, because the neural networks are used as the black-box tool, it is difficult to add new functions to already designed networks. Nakamura and Sekiguchi tried to develop the information processing system using chaotic dynamics[6, 7]. They designed the control algorithm for the mobile robot using entrainment and synchronization of the robot dynamics and environment dynamics based on Arnold differential equation.

In this paper, we develop the dynamics based information processing system to handle the humanoid whole body motion. The nonlinear dynamics memorizes, generates and transits humanoid whole body motions based on the entrainment and detrainment phenomenon with polynomial representation. Some design methods of the nonlinear dynamics that has an attractor using the Lyapunov function have been proposed [8]~[10]. In our method, the dynamics is defined as the vector field in the N dimensional space and it has an attractor to any closed curved line. By changing the vector field configuration, the dynamics behavior is changeable.

Using the proposed methods of the motion reduction and dynamics based information processing system, we integrate the whole body motion generator for the humanoid robot with 20 degree-of-freedom.

2 Motion reduction and symbolization

The reduction of the whole body motion means the symbolization of the motion. By the mapping function and its inverse function, the whole body motion is reduced to the symbol and restored from symbol. Consider the humanoid robot with n degree-of-freedom. The humanoid motion data Y is defined as follows.

$$Y = \begin{bmatrix} y_1[1] & y_1[2] & \cdots & y_1[m] \\ y_2[1] & y_2[2] & \cdots & y_2[m] \\ \vdots & \vdots & & \vdots \\ y_n[1] & y_n[2] & \cdots & y_n[m] \end{bmatrix} \quad (1)$$

For example, $y_i[k]$ menas the angles of each joints. By the singular value decomposition of Y

$$Y = USV^T \quad (2)$$

$$U = [\ U_1 \mid U_2\ ] \tag{3}$$
$$S = \left[\begin{array}{c|c} S_1 & \\ \hline & S_2 \end{array}\right] \tag{4}$$
$$S_1 = \mathrm{diag}\{\ s_1 \quad s_2 \quad \cdots \quad s_r\ \} \tag{5}$$
$$S_2 = \mathrm{diag}\{\ s_{r+1} \quad s_{r+2} \quad \cdots \quad s_n\ \} \tag{6}$$
$$V^T = \left[\begin{array}{c} V_1^T \\ \hline V_2^T \end{array}\right] \tag{7}$$

if $s_r \gg s_{r+1}$ is satisfied, Y is reduced to the r dimensional motion V_1^T as follows.

$$Y = FV_1^T \tag{8}$$
$$F = U_1 S_1 \tag{9}$$

Here, $U_1 \in \boldsymbol{R}^{n\times r}$, $V_1^T \in \boldsymbol{R}^{r\times m}$. F is the mapping function for the symbol and whole body motion. This result shows that

1. The whole body motion Y that represents a curved line in n dimensional space, is symbolized by a curved line V_1^T in the r dimensional space. If $y_i[k]$ ($i = 1, \cdots n,\ k = 1, \cdots m$) is the periodic sequence, Y and V_1^T mean the closed curved lines M and C respectively.
2. By checking the singular value s_i ($i = 1, 2, \cdots, n$), the appropriate r is selected.
3. The first column vector of V_1 is the principal component of the motion Y, the second column vector is the second principal component.
4. The inverse function of F is $S_1^{-1}U_1^T$

3 Design of the dynamics based information processing system

3.1 Dynamics and brain-like information processing system

The human does not remember the time sequence data of the joint angle respect to the whole body motion. The motions are symbolized and selected appropriately based on the internal state of our brain and input sensor signal. We explain this process and phenomenon corresponding to the dynamics. Consider the following discrete time dynamic equation.

$$\boldsymbol{x}[k+1] = \boldsymbol{x}[k] + \boldsymbol{g}(\boldsymbol{u}[k], \boldsymbol{x}[k]) \tag{10}$$

$\boldsymbol{x} \in \boldsymbol{R}^N$ is the state vector and $\boldsymbol{u} \in \boldsymbol{R}^L$ is the input signal. $\boldsymbol{x}[k]$ moves in the N dimensional space. If this dynamics has attractor to one closed curved line C, $\boldsymbol{x}[k]$ is entrained to C with an initial condition $\boldsymbol{x}_0$ with $\boldsymbol{u}[k] = 0$. Suppose that the C corresponds to the reduced joint angle V_1^T in equation (8). C means the symbol of the humanoid motion M. The time sequence data of $\boldsymbol{x}[k]$ yields that of humanoid joint angles by mapping function F in equation (9), which means the dynamics memorizes and generates the humanoid whole body motion.

Suppose the dynamics in equation (10) has some attractors and transits to each attractors by the input signal $\boldsymbol{u}[k]$, which means the motion transition of the humanoid robot based on the input sensor signal.

In this section, we design the dynamical system that has some attractors C and transits to each attractors to handle the humanoid motions M.

3.2 Design of the nonlinear dynamics

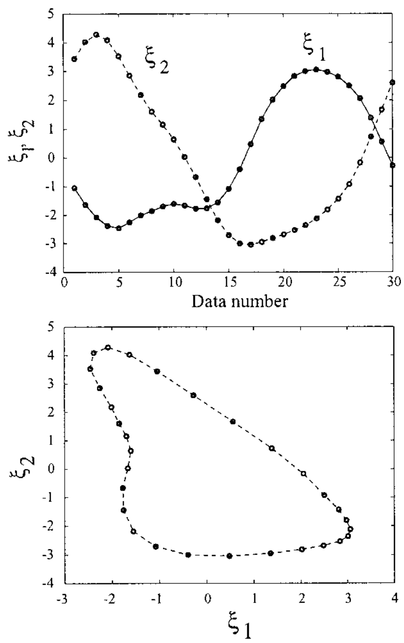

Figure 1: Time sequence data and closed curved line

Consider the reduced whole body motion in N degree-of-freedom. The time sequence data of each degree-of-freedom is set as $\xi_i[k]$ ($i = 1, \cdots N,\ k = 1, \cdots m$). Assuming the time sequence data is periodic, $\boldsymbol{\xi}[k]$

$$\boldsymbol{\xi}[k] = [\ \xi_1[k] \quad \xi_2[k] \quad \cdots \quad \xi_N[k]\ ]^T \tag{11}$$

consists the closed curved line Ξ in the N dimensional space.

$$\Xi = [\ \boldsymbol{\xi}[1] \quad \boldsymbol{\xi}[2] \quad \cdots \quad \boldsymbol{\xi}[m] \quad \boldsymbol{\xi}[1]\ ] \tag{12}$$

Figure 1 shows the simple case of $N = 2$. Two time sequences $\xi_1[k]$, $\xi_2[k]$ construct the closed curved line in the 2 dimensional space. In this paper, we design a nonlinear dynamics $\mathcal{D}$ that has attracter to this closed curved line with the following formulation.

$$\mathcal{D}\ :\ \boldsymbol{x}[k+1] = \boldsymbol{x}[k] + \boldsymbol{f}(\boldsymbol{x}[k]) \tag{13}$$

The design algorism is as follows.

Step 1 Draw the closed curved line C in equation (12) in the N dimensional space.

Step 2 Define the vector field $\boldsymbol{f}(\boldsymbol{\eta}_i)$ on the point $\boldsymbol{\eta}_i$ which is in domain D in the N dimensional space according to the following algorism (refer to figure 2).

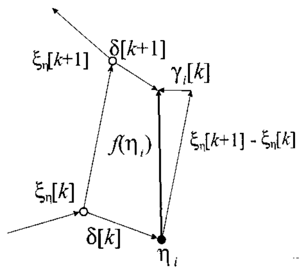

Figure 2: Vector definition

$$\boldsymbol{f}(\boldsymbol{\eta}_i) = (\boldsymbol{\xi}_\eta[k+1] - \boldsymbol{\xi}_\eta[k]) + \boldsymbol{\gamma}_i[k] \quad (14)$$

$$\boldsymbol{\xi}_\eta[k] = \arg\min_{\boldsymbol{\xi}[k] \ \ k} \|\boldsymbol{\eta}_i - \boldsymbol{\xi}[k]\| \quad (15)$$

Because $\boldsymbol{\eta}_i = \boldsymbol{\xi}_\eta[k] + \boldsymbol{\delta}[k]$, the sufficiency of that the closed curved line becomes the attractor of the dynamics is

$$\|\boldsymbol{\delta}[k] + \boldsymbol{\gamma}_i[k]\| < \|\boldsymbol{\delta}[k+1]\| \quad (16)$$

By satisfing this condition, $\boldsymbol{\delta}[k] \to 0$ at $k \to \infty$.

Step 3 On the points $\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \cdots, \boldsymbol{\eta}_m$ in the domain D, define the vectors $\boldsymbol{f}(\boldsymbol{\eta}_1), \boldsymbol{f}(\boldsymbol{\eta}_2), \cdots, \boldsymbol{f}(\boldsymbol{\eta}_m)$ shown in figure 3

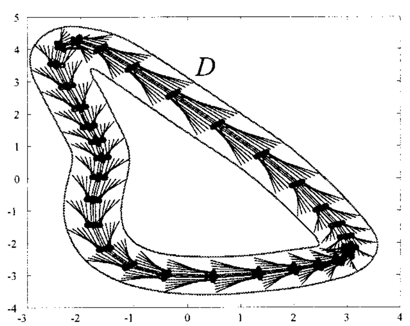

Figure 3: Definition of the vector field

Step 4 Obtain the nonlinear function $\boldsymbol{f}(\boldsymbol{x}[k])$ that approximate $\boldsymbol{f}(\boldsymbol{\eta}_i)$ by using the polynomial approximation of $\boldsymbol{x}_i$ as follows.

$$\boldsymbol{f}(\boldsymbol{\eta}_i) = \sum_{P=0}^{l} \sum_{\substack{p_1, \cdots, p_n \\ \sum p_k = P \\ p_k \,:\, \text{positive integer}}}^{l} a_{(p_1 p_2 \cdots p_n)} \prod_{j=1}^{n} \eta_{ij}^{p_j} \quad (17)$$

$$\boldsymbol{\eta}_i = \begin{bmatrix} \eta_{i1} & \eta_{i2} & \cdots & \eta_{iN} \end{bmatrix}^T \quad (18)$$

a_{ij}^{pq} are constants. It is easy to calculate $\boldsymbol{f}(\boldsymbol{x}[k])$ by the least square method as follows.

$$\Phi(a_{pq}^{ij}) = FH^{\#} \quad (19)$$

$$F = \begin{bmatrix} \boldsymbol{f}(\boldsymbol{\eta}_1) & \boldsymbol{f}(\boldsymbol{\eta}_2) & \cdots & \boldsymbol{f}(\boldsymbol{\eta}_m) \end{bmatrix} \quad (20)$$

$$H = \begin{bmatrix} \eta_{11}^{\ell} & \eta_{21}^{\ell} & \cdots & \eta_{m1}^{\ell} \\ \vdots & & & \\ \eta_{1N}^{\ell} & \eta_{2N}^{\ell} & \cdots & \eta_{mN}^{\ell} \\ \eta_{11}^{\ell-1}\eta_{12} & \eta_{21}^{\ell-1}\eta_{22} & \cdots & \eta_{m1}^{\ell-1}\eta_{m2} \\ \vdots & & & \\ 1 & 1 & \cdots & 1 \end{bmatrix} \quad (21)$$

Φ is the constant matrix. The domain D is the attracted region, which means the point $\boldsymbol{x}[k]$ inside D is entrained to C at $k \to \infty$. If Φ gives the right approximation of the defined vector field, we obtain the nonlinear dynamics that has an attractor to the closed curved line C. Figure 4 shows the motion of the designed nonlinear dynamics. '+' means the initial condition of $\boldsymbol{x}[0]$. It is entrained to C. This result shows that the designed dynamics memorizes the time sequence data $\boldsymbol{\xi}[k]$ $(k = 1, 2, \cdots m)$ and generate the whole body motion.

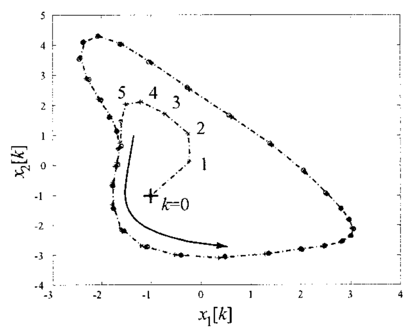

Figure 4: Motion of the nonlinear dynamics

3.3 Set of the input signal

The designed dynamics $\mathcal{D}$ in equation (13) moves autonomously. From the initial state condition $\boldsymbol{x}[0]$ in the domain D, it converges to C. In this section, we set the input signal to the designed dynamics. By adding another time sequence $\boldsymbol{u}[k]$ $(\in \boldsymbol{R}^L, \ k = 1, 2, \cdots, M)$, the dynamics is represented as follows.

$$\widehat{\mathcal{D}} \ : \ \boldsymbol{x}[k+1] = \boldsymbol{x}[k] + \boldsymbol{g}(\boldsymbol{u}[k], \boldsymbol{x}[k]) \quad (22)$$

By changing Ξ in equation (12) as

$$\Xi = \begin{bmatrix} \boldsymbol{\mu}[1] & \boldsymbol{\mu}[2] & \cdots & \boldsymbol{\mu}[M] & \boldsymbol{\mu}[1] \\ \boldsymbol{\xi}[1] & \boldsymbol{\xi}[2] & \cdots & \boldsymbol{\xi}[M] & \boldsymbol{\xi}[1] \end{bmatrix} \quad (23)$$

$\boldsymbol{\mu}[k]$: given

$g(u[k], x[k])$ is calculated by the same algorism as $f(x[k])$. Figure 5 shows the motion of the dynamics with 1 dimensional input $u[k]$. The input $u[k]$ is changed from

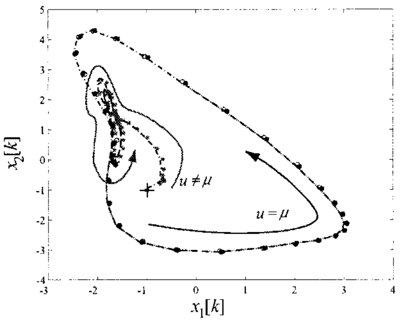

Figure 5: Motion of the nonlinear dynamics with the input signal

one signal to $\mu[k]$ in time $k = \widehat{k}$. The dynamics is entrained to C after going around in the space according to the change of input. This result means that the input signal changes the structure of the nonlinear dynamics by changing the vector field $g(u[k], x[k])$, and entrains the dynamics to C.

3.4 Dynamics with multi attractors

We modify the dynamics $\widehat{D}$ so that it has some attractors and transits to each attractors. The nonlinear dynamics is re-written as follows.

$$\widehat{\mathcal{D}}^w \ : \ x[k+1] = x[k] + w(x[k])g(u[k], x[k]) \quad (24)$$

$w(x[k])$ is the weighting function defined as follows.

$$w(x[k]) = 1 - \frac{1}{1 + \exp\{a(\omega(x[k]) - 1)\}} \quad (25)$$

$$\omega(x[k]) = (x^T[k] - X_0^T)Q(x[k] - X_0) \quad (26)$$

a is a constant. Q and X_0 define the following ellipsoid E.

$$(x^T[k] - X_0^T)Q(x^T[k] - X_0) = 1 \quad (27)$$

X_0 is the center of the ellipsoid, Q is the positive definite metrics. Equation (25) means that if $x[k]$ is inside the ellipsoid in equation (27), the weighting function is 1. Based on the dynamics $\widehat{\mathcal{D}}_i^w$ ($i = 1, 2, \cdots$), we design the nonlinear dynamics $\widetilde{\mathcal{D}}$ which has some attractors as follows.

$$\widetilde{\mathcal{D}} \ : \ x[k+1] = x[k] + \sum_i w_i(x[k])g_i(u[k], x[k]) \quad (28)$$

This configuration means that the vector field that defines an attractor is effective in the ellipsoid. One vector field defined by $g_i(u[k], x[k])$ dose not have influence to other attractors. Because the vector fields is defined by the sum of $g_i(u[k], x[k])$ surrounded by the ellipsoid E_i, it is easy to add the new attractor. The state $x[k]$ moves to some attractors in term of the input signal $u[k]$. Figure 6 shows the designed dynamics that has three attractors in the 2 dimensional space. By the 1 dimensional input

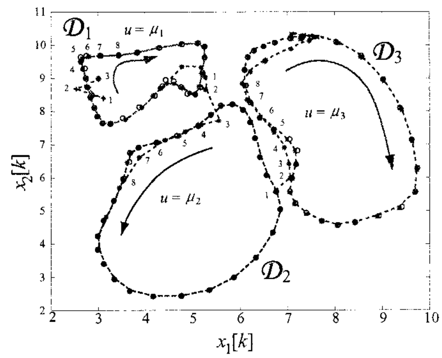

Figure 6: Designed three attractors and dynamics

$u[k]$, the dynamics moves in the space attracted to the closed curved lines.

3.5 Recognition of self motion

The designed dynamics in equation (22) is modified as follows.

$$\bar{\mathcal{D}} \ : \ \begin{bmatrix} \widehat{u}[k+1] \\ x[k+1] \end{bmatrix} = \begin{bmatrix} u[k] \\ x[k] \end{bmatrix} + h(X[k]) \quad (29)$$

The vector field h is calculate by the same ways as f and

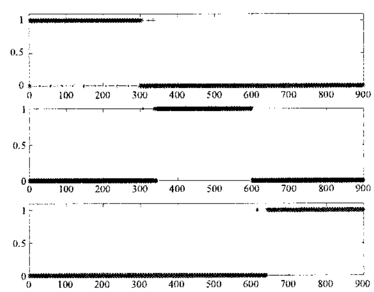

Figure 7: Recognition of the input signal

g. Because $\widehat{u}[k+1]$ is the prediction of the $u[k+1]$, this dynamics recognizes which closed curved line the state vector is attracted by comparing $u[k+1]$ and $\widehat{u}[k+1]$. Figure 7 shows the recognition index of the attraction. The vertical axis is the recognition index (RI) that is defined

$$RI_i = \begin{cases} 1 & (\|\widehat{u}[k+1] - u[k+1]\| \geq \alpha) \\ 0 & (\|\widehat{u}[k+1] - u[k+1]\| < \alpha) \end{cases} \quad (30)$$

The upper figure, the middle figure and the lower figure show the recognition of $u_i[k]$ (correspond to $\mathcal{D}_i$,

$i = 1, 2, 3$) respectively. By checking RI_i, the humanoid recognizes which motion it takes.

4 Generation of the whole body motion

4.1 Whole body motion of the humanoid robot

In this section, we design the humanoid whole body motion using the dynamics based information processing system. Figure 8 shows the humanoid robot (FUJITSU Humanoid HOAP-1) that has 20 degree of freedom. We

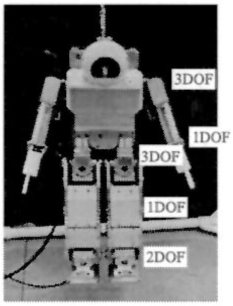

Figure 8: Humanoid robot HOAP-1

design the "walk" motion and "squat" motion. Figure 9 shows the original motion. This humanoid robot is not grounded. Because the dynamic based information processing system yields only the time sequence of the joint angle, the feedback controller that stabilizes each motions should be implemented.

4.2 Design of the dynamics

From the "walk" motion $Y_w \in \boldsymbol{R}^{20}$ and "squat" motion $Y_s \in \boldsymbol{R}^{20}$, we obtain the reduced motions $V_w^T \in \boldsymbol{R}^3$, $V_s^T \in \boldsymbol{R}^3$ in three dimensional space.

Based on the reduced motion, we design the dynamics based on the following equation

$$\boldsymbol{x}[k+1] = \boldsymbol{x}[k] + \sum_{i=w,s} w_i(\boldsymbol{x}[k])\boldsymbol{f}_i(\boldsymbol{x}[k]) + \sum_{i=w,s} K_i O_i(\boldsymbol{x}[k]) \tag{31}$$

$$O_i(\boldsymbol{x}[k]) = \delta(X_i^c - \boldsymbol{x}[k]) \tag{32}$$

By changing K_w and K_s, the humanoid transits its motion. δ is constant. X_w^c and X_s^c mean the center of reduced closed curved line "walk" and "squat" respectively. Figure 10 shows the motion of the dynamics. From the initial position, the dynamics is entrained to the walk motion (arrow 1), entrained to squat motion (arrow 2) and finally entrained to walk motion again (arrow 3).

4.3 Motion of the humanoid robot

Figure 11 shows the generated humanoid motion. Because while the dynamics is attracted to the closed curved

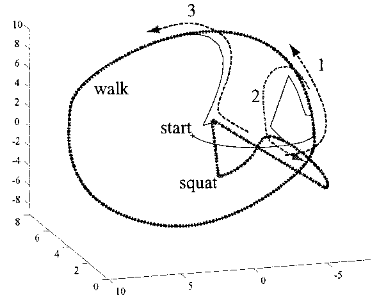

Figure 10: Motion of the dynamics

line, the humanoid motion is as same as figure 9, and only the transition motions are shown. The continuous transition is generated.

5 Conclusions

In this paper, we propose the motion reduction method and the brain-like information processing that realizes the memorization and generation of the humanoid whole body motion using the nonlinear dynamics with the polynomial configuration. The results of this paper are as follows.

1. We propose the motion reduction method using the principal component analysis based on singular value decomposition.
2. We propose the design method of the nonlinear dynamics that has an attractor to the N dimensional closed curved line with polynomial configuration.
3. Using the proposed method, the whole body humanoid motion is generated.

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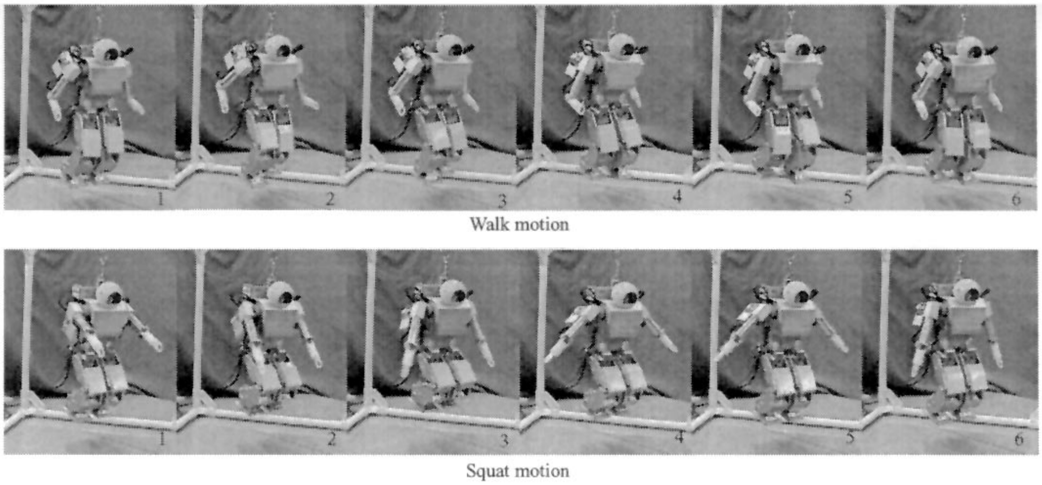

Figure 9: Humanoid motion

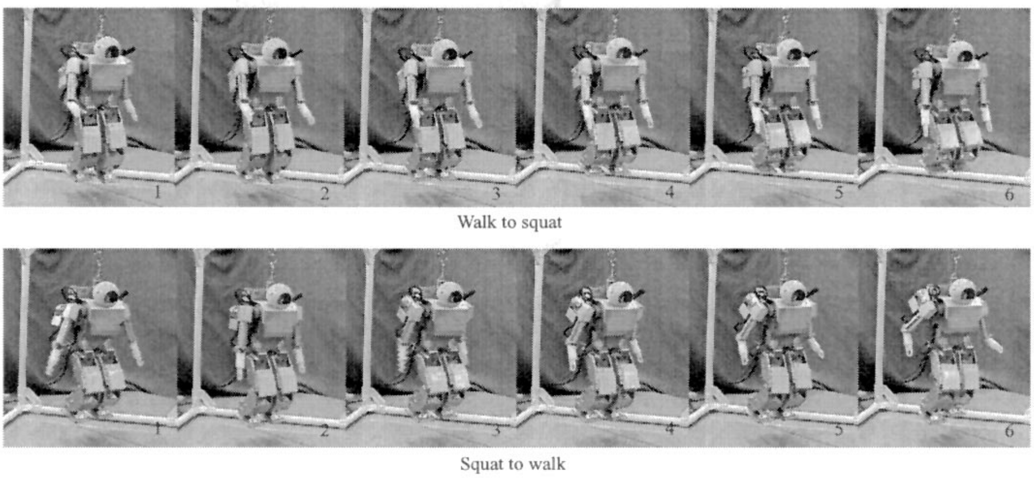

Figure 11: Motion transintion of the humanoid robot

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