

# Motion Emergency of Humanoid Robots by an Attractor Design of a Nonlinear Dynamics

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**Abstract**—The human motions are generated through the interaction between the body and its environments. The information processing system defines the current motion using the signal feedback of the body state and environments. The motion pattern dose not exists a priori but emerges as the result of the entrainment phenomenon for the dynamics of the information processing, the human body and its environments. In this paper, based on the dynamics-based information processing system, we propose the motion emergency system design method for a humanoid robot designing a dynamical system that has an attractor considering the robot body dynamics. From the control engineering point of view, the proposed method designs a controller that stabilizes the robot to an equilibrium trajectory.

**Index Terms**—motion emergency, attractor design, humanoid robot, dynamics-based information processing, nonlinear dynamics

## I. INTRODUCTION

For a motion control of robots, the motion pattern and a controller are designed and the robot is controlled so that it strictly follows to the motion pattern as shown in Figure 1. The motion pattern is designed satisfying

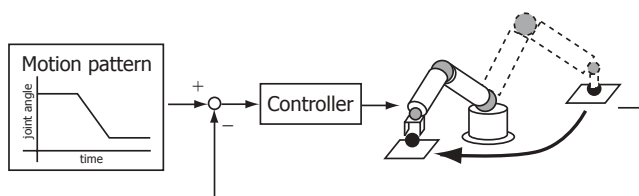


Fig. 1. Motion control of robots

the dynamical constraint (e.g. torque or angular velocity limitation) of the robot and the controller is designed so that it has robustness for the model perturbations and disturbances. For the change of the robot motion, multiple motion patterns are prepared and switched to one another. This method is useful for the industrial robots that require the precision control for a task execution. The same control way has been employed, so far, for humanoid robots. The motion patterns are designed considering ZMP (Zero Moment Point) and COG (Center of Gravity) for

the stable motion, and the controllers are designed that realize the strict reference following and robust stability. The drawbacks of this control method are as follows.

- 1) The robot realizes one motion with one motion pattern and one controller. For the change of motion, it requires the change of motion pattern and controller.
- 2) Because the change of motion pattern and/or controller is a discrete event, the robot changes the motion discontinuously or gets through the initial position in every motion change.
- 3) Because the motion pattern has time stamp, it goes ahead regardless of the robot state, which means the robot possibly takes a walk motion patters in spite of its falling down.

On the other hand, the human motions are generated through the interaction between the body and its environments. As shown in Figure 2, the information processing system defines the current motion using the signal feedback of the body state and the environments. This closed loop system yields the entrainment phenomenon that causes the motion generation. Because of the entrain-

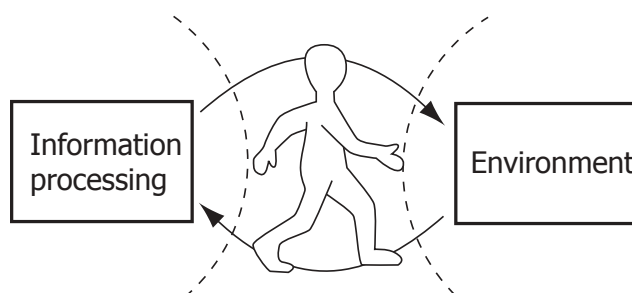


Fig. 2. Motion generation of the human

ment phenomenon, the human motion has robustness for the change of the environment and disturbances, and the motion change is realized continuously. This consideration leads that the motion pattern dose not exist a priori but emerges as the result of the entrainment phenomenon for the dynamics of the information processing, the human body and its environments, which means the motion change is not caused by the change of the motion pattern but the

change of the information processing and/or environments. This concept coincides to the "embodiment" that represents the close relationship between the intelligence and the body. For the design of the intelligent and autonomous robot, it is necessary to realize the motion emergency of the robots based on the interaction between the information processing (controller) and the environment through the robot body[1].

Some researches on the motion control using the entrainment phenomenon of the dynamics have been reported. Ijspeert proposed the movement imitation system based on CPG (Central Pattern Generator) [2], which aims at using the entrainment phenomenon of the CPG. Kotosaka designed a rhythmical motion using CPG[3]. We have developed the design method of the dynamical system that has an attractor on the closed curved line in  $N$  dimensional space, and proposed the dynamics-based information processing system that memorizes and reproduces the humanoid whole body motion. The purpose of these methods is to design the motion pattern generator using the entrainment phenomenon of the dynamical system, however the designed systems separate off the environments. Tani designed the robot navigation system using the recurrent neural network and showed the capability of the entrainment phenomenon for the design of the symbol emergency and symbol manipulation system[5]. Tsujita realized a CPG based walking motion of a quadruped locomotion robot using the feedback of the sensor signals from the environment[6]. Because these methods utilize the existing dynamical system, the analysis is qualitative and phenomenological, and the design requires a parameter adjustment with a trial and error.

In this paper, based on the dynamical system design method in [4], we realize the motion emergency system for a humanoid robot designing a dynamical system that has an attractor considering the robot body dynamics. From the control engineering point of view, the conventional method designs a controller that stabilizes the robot to an equilibrium point and a trajectory of the equilibrium point, on the other hand, the proposed method designs a controller that stabilizes the robot to an equilibrium trajectory.

## II. DYNAMICAL SYSTEM DESIGN METHOD

### A. Dynamics and the whole body motion

First, I will illustrate the dynamics-based information processing system in [4]. Consider the cyclic whole body motion  $\mathcal{M}$  of the robot with  $N$  joints. The posture (joint angles)  $\theta[k]$  in time  $k$  represents one point in  $N$  dimensional joint space. The data set  $M$  of the whole body motion  $\mathcal{M}$  defined as follows

$$M = [ \theta[1] \ \theta[2] \ \cdots \ \theta[m] ] \quad (1)$$

consists of the time sequence data of  $\theta[k]$  and draws a closed curved line  $C$  as shown in Figure 3, where  $m$  means the number of data.

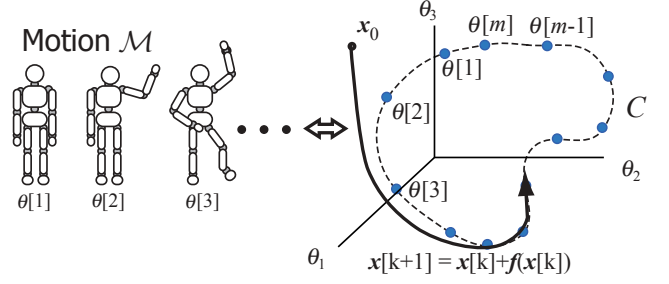


Fig. 3. The robot posture, motion in the joint space

On the other hand, consider the discrete time dynamics as shown in the following difference equation.

$$x[k+1] = x[k] + f(x[k]) \quad (2)$$

Suppose that this dynamics has an attractor on the closed curved line  $C$ , which means the state vector  $x[k]$  starting from the initial value  $x_0$  converges to the following equation

$$\lim_{k \rightarrow \infty} x[k] = \theta[k+k_0] \quad (3)$$

where  $k_0$  depends on  $x_0$ . In this case, the dynamics memorizes and reproduces the time sequence data of the whole body motion  $\mathcal{M}$ .

### B. Dynamics-based information processing

Because the second term  $f(x[k])$  in the right hand side of equation (2) represents the vector field in  $x$ -space, the dynamics is designed by the functional approximation of the defined vector field. Set the points  $\eta_j^k$  around  $\theta[k]$  as

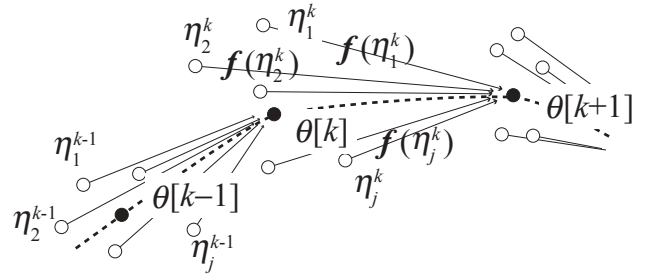


Fig. 4. Definition of the vector field

shown in Figure 4 and define the vector  $f(\eta_j^k)$  on each  $\eta_j^k$  such that

$$f(\eta_j^k) = \theta[k+1] - \eta_j^k \quad (4)$$

By defining  $f(x[k])$  as the following polynomial function,

$$f(x[k]) = \Theta \phi(x[k]) \quad (5)$$

where  $\Theta$  is the coefficient matrix of the polynomial and  $\phi(x[k])$  consists of the power of the elements of  $x[k]$ ,  $\Theta$  is obtained by the following least square method.

$$\Theta = F \Phi^\# \quad (6)$$

$$F = [ f(\eta_1^1) \ \cdots \ f(\eta_j^1) \ \cdots \ f(\eta_j^k) \ \cdots ] \quad (7)$$

$$\Phi = [ \phi(\eta_1^1) \ \cdots \ \phi(\eta_1^j) \ \cdots \ \phi(\eta_j^k) \ \cdots ] \quad (8)$$

In this method, because the dynamics in equation (2) is in the virtual space, which means the robot body dynamics and the environments are not considered, the designed dynamics works as the information processing system that yields a motion pattern. For the stable motion emergency or control, the robot body dynamics has to be considered.

### C. Attractor design considering the physical dynamics

Based on the dynamics design method in section II-B, we develop the motion emergency system. For simplicity, we show the dynamics design method with a linear system. Consider the linear system represented by the following difference equation.

$$\mathbf{x}[k+1] = A\mathbf{x}[k] + B\mathbf{u}[k], \quad \mathbf{x}[k] \in \mathbf{R}^N \quad (9)$$

where  $\mathbf{x}[k]$  is the state vector,  $\mathbf{u}[k]$  is the input. Assume that this system is controllable, which means

$$\text{rank} \left( \begin{bmatrix} B & AB & \cdots & A^{N-1}B \end{bmatrix} \right) = N \quad (10)$$

is satisfied. Assume that the motion pattern  $\Xi$  of the state vector  $\mathbf{x}[k]$  is given as

$$\Xi = [ \xi[1] \quad \xi[2] \quad \cdots \quad \xi[m] ] \quad (11)$$

that is assumed to satisfy the physical constraint of equation (9), which means the input signal  $\mathbf{u}[k]$  exists that realizes the motion  $\Xi$ . Consider the point  $\boldsymbol{\eta}^k[0]$  located near  $\xi[k]$  as shown in Figure 5. When the matrix  $B$  in equation (9) is not

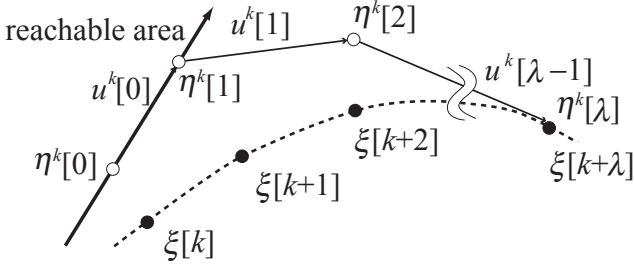


Fig. 5. Definition of the vector field for physical dynamics

square or nonsingular matrix, the span of  $B$  restricts the reachable area of  $\boldsymbol{\eta}^k[1]$ , which means we can not define the vector field  $\mathbf{f}(\boldsymbol{\eta}^k[0])$  in equation (4). Consequently, consider the multi-steps convergence that means  $\boldsymbol{\eta}^k[0]$  coincides to  $\xi[k+\lambda]$  in  $\lambda$ -step ahead. If the linear system in equation (9) is controllable,  $\mathbf{u}^k[\ell]$  ( $\ell = 0, 1, \dots, \lambda-1$ ) that satisfies

$$\boldsymbol{\eta}^k[\lambda] = \xi[k+\lambda] \quad (12)$$

exists with  $\lambda \geq N$ .  $\mathbf{u}^k[\ell]$  and  $\boldsymbol{\eta}^k[\ell]$  are calculated as follows. The point  $\boldsymbol{\eta}^k[\lambda]$  is represented by the following equation.

$$\boldsymbol{\eta}^k[\lambda] = A\boldsymbol{\eta}^k[\lambda-1] + B\mathbf{u}^k[\lambda-1] \quad (13)$$

$$= A^\lambda \boldsymbol{\eta}^k[0] + \Gamma U \quad (14)$$

$$\Gamma = \begin{bmatrix} B & AB & \cdots & A^{\lambda-1}B \end{bmatrix} \quad (15)$$

$$U = \begin{bmatrix} \mathbf{u}^{kT}[\lambda-1] & \mathbf{u}^{kT}[\lambda-2] & \cdots & \mathbf{u}^{kT}[0] \end{bmatrix}^T \quad (16)$$

From these relationships, we obtain  $\mathbf{u}^k[\ell]$  by the following equation.

$$U = \Gamma^\# (\xi[k+\lambda] - A^\lambda \boldsymbol{\eta}^k[0]) \quad (17)$$

where  $\Gamma^\#$  exists because  $\Gamma$  is an extended controllable matrix. Based on the obtained  $\mathbf{u}^k[\ell]$ , we can calculate  $\boldsymbol{\eta}^k[\ell]$ . By setting some points  $\boldsymbol{\eta}_j^k[0]$  ( $k = 1, 2, \dots, m$ ,  $j = 1, 2, \dots$ ) as in section II-B, we obtain the data set of  $\boldsymbol{\eta}_j^k[\ell]$  and  $\mathbf{u}^k[\ell]$  and calculate the function

$$\mathbf{u}[k] = \mathbf{f}(\mathbf{x}[k]) = \Theta\phi(\mathbf{x}[k]) \quad (18)$$

by the same way as in section II-B using the polynomial function approximation.

For the nonlinear system, obtain the linear approximate system

$$\mathbf{x}[k+1] = A_i \mathbf{x}[k] + B_i \mathbf{u}[k] \quad (19)$$

around  $\xi[i]$ , and obtain  $\mathbf{u}^k[\ell]$  by changing equation (17) to

$$U = \hat{\Gamma}^\# \left( \xi[k+\lambda] - \prod_{i=k}^{k+\lambda-2} A_i \boldsymbol{\eta}^k[0] \right) \quad (20)$$

$$\hat{\Gamma} = \begin{bmatrix} B_{k+\lambda-1} & \cdots & \prod_{i=k}^{k+\lambda-3} A_i B_k \end{bmatrix} \quad (21)$$

The following considerations are necessary for the proposed method.

#### Selection of $\lambda$

The parameter  $\lambda$  have to be selected such that  $\lambda \geq N$  and defines the convergence speed of  $\mathbf{x}[k]$  to the attractor. The selection of  $\lambda$  corresponds to the set of the feedback gain.

#### Minimization of the input signal

In equation (17), the pseudo inverse of  $\Gamma$  is utilized, which is not approximation but the minimization of the input signal  $\mathbf{u}[k]$  (to be exact, minimization of  $\|U\|$ ).

#### Existence of $\hat{\Gamma}^\#$

In equation (20) the existence of  $\hat{\Gamma}^\#$  can not be guaranteed unfortunately. However, when the nonlinear system is reachable (for nonlinear system, not controllable but reachable),  $\hat{\Gamma}^\#$  is existable.

#### Accuracy of the linear approximation

When the distance between  $\boldsymbol{\eta}^k[\ell]$  and  $\xi[k+\ell]$  is not small, the accuracy of the linear approximation is not sufficient. In this case, the iteration method such that  $A_i$  and  $B_i$  are recalculated around  $\boldsymbol{\eta}^k[\ell]$  is necessary.

#### Structure of the controller

The controller in equation (18) working as the information processing in Figure 2, is the nonlinear state feedback controller. By the linear state feedback, the dynamics is stabilized around to an equilibrium point. Because of the nonlinearity of the controller, the equilibrium trajectory is realized.

### Meaning of $\Xi$

The existence of  $\Xi$  is assumed. However, the motion trajectory of the dynamics does not always coincide to  $\Xi$  because of the high torque requirement over the limitation or model uncertainty.  $\Xi$  only plays a role of the 'seed' for robot motion.

### III. NUMERICAL EXAMPLE OF THE PROPOSED METHOD

For the evaluation of the proposed method, we show a simple numerical example. Consider the following second order dynamical system.

$$m\ddot{x} = kx - d\dot{x} + u \quad (22)$$

This is the mass ( $m[\text{kg}]$ ), spring ( $k[\text{N/m}]$ ), damper ( $d[\text{Ns/m}]$ ) system with a negative spring (the spring constant is smaller than zero), which means it is an unstable system. For this system, obtaining the discrete time system by the euler approximation, we set the motion trajectory of  $x[k]$  as shown in the left hand side of Figure 6 that draws the closed curved line  $C$  in

$$\mathbf{x}[k] = [x[k] \quad \dot{x}[k]]^T \quad (23)$$

space as shown in the right hand side of Figure 6. According to the design method of the controller in equa-

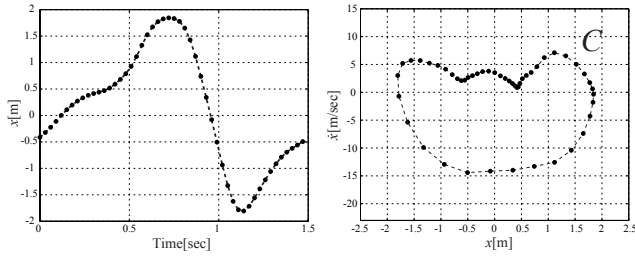


Fig. 6. Motion trajectory and closed curved line

tion (18), we set  $\eta_j^k[0]$  ( $k = 1, 2, \dots, m$ ,  $j = 1, 2, \dots$ ) and calculate  $u^k[\ell]$  and  $\eta_j^k[\ell]$  ( $\ell = 1, 2, \dots, \lambda$ ). Figure 7 shows  $\eta_j^k[0]$  and  $\eta_j^k[\ell]$  represented by '\*' and 'o' respectively. It

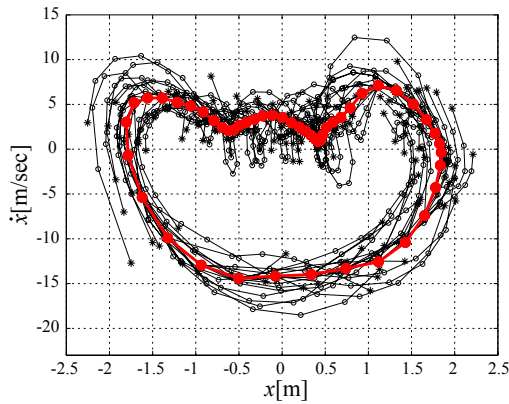


Fig. 7. Defined vector field

is clear that the dynamical constraints are satisfied in this

vector field (relationship between the position  $x[k]$  and the velocity  $\dot{x}[k]$ ).  $u[k]$  is approximated by the seventh order polynomial of  $x[k]$ . Figure 8 shows the motion of the state vector  $\mathbf{x}[k]$  from some initial points. The mark '+' means

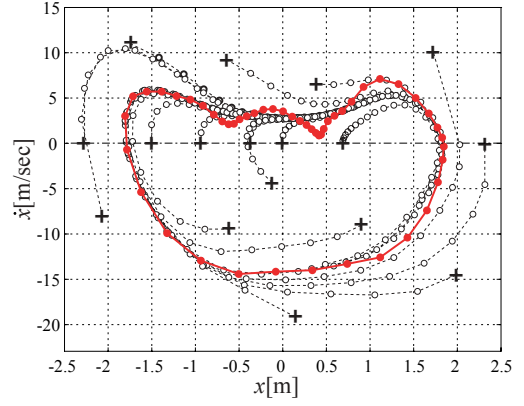


Fig. 8. Motion of the controlled dynamics

the initial points. They are entrained to the closed curved line. The initial points on the  $x$ -axis stopped at first and start to move by the input signal. Figure 9 shows the vector field of the dynamics without a controller (the left hand side) and with a controller (the right hand side). The lengths

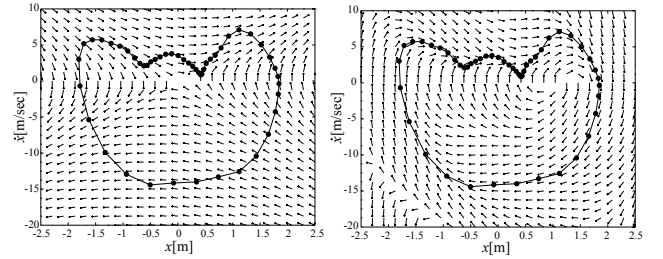


Fig. 9. Designed vector field

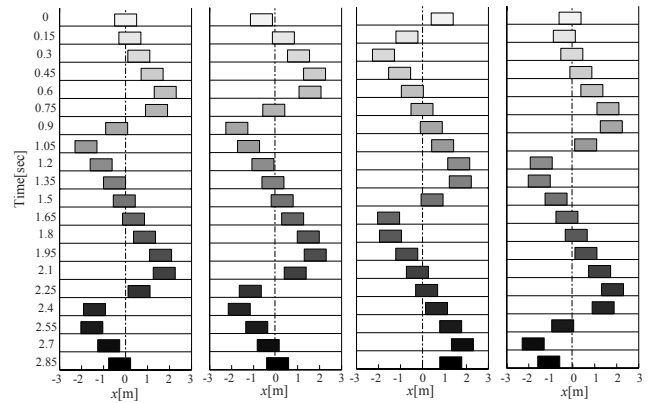


Fig. 10. Motions of the mass

of the arrows are normalized. These figures show that the controller stabilizes the system so that the closed curved line becomes the equilibrium trajectory. Figure 10 shows the motions of the mass from the different initial points. The motions of each masses are same though the phases

of the motions are different, this is because the controller decides the input signal for the stabilization and the motion generation depending on the information obtained from the environment (state of the mass).

#### IV. MOTION EMERGENCY OF THE HUMANOID ROBOT

In this section, we develop the motion emergency system for the humanoid robot shown in Figure 11 in the real world. The dynamics of the humanoid robot is represented

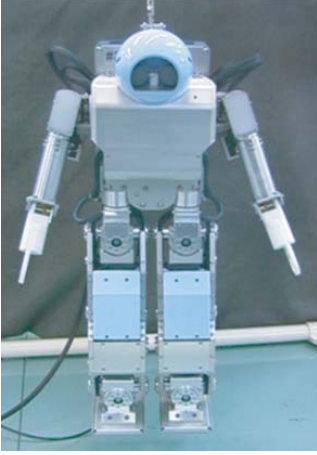


Fig. 11. Humanoid robot HOAP-1

by the following equations based on the inverted pendulum model[7].

$$M\ddot{x}_G = K(x_G - x_z) \quad (24)$$

$$M\ddot{y}_G = K(y_G - y_z) \quad (25)$$

$$K = \frac{M(\ddot{z}_G + g)}{z_G - z_z} \quad (26)$$

where  $X_G = [x_G \ y_G \ z_G]^T$  is the position of COG,  $X_z = [x_z \ y_z \ z_z]^T$  is the position of ZMP,  $M$  means the weight of the robot and  $g$  is the gravity. By setting the state vector  $x$  as

$$x = [X_G^T \ \dot{X}_G^T]^T \quad (27)$$

we obtain the state space representation of the humanoid body dynamics as

$$\dot{x} = f(x) + h(x, X_z(\theta)) \quad (28)$$

where the input of the robot  $X_z$  that is calculated from the angular acceleration of the whole body joints. The joint angles are controlled so that the COG is entrained to the reference trajectory based on the COG jacobian[8].

Here we consider the squat motion of the humanoid robot. The reference trajectory of the COG (position and velocity) and the knee joints (joint angle and angular velocity) are given, and the controller

$$\theta[k+1] = \Theta\phi \left( [X_G^T[k] \ \theta^T[k]]^T \right) \quad (29)$$

is calculated by the polynomial structure as in equation (18). For simplicity, the upper body joints are fixed and the hip joints are utilized to control the ZMP.

Figure 12 shows the trajectory of the knee joint  $\theta_{knee}[k]$  and  $x_G[k]$ . The solid lines show the reference trajectories,

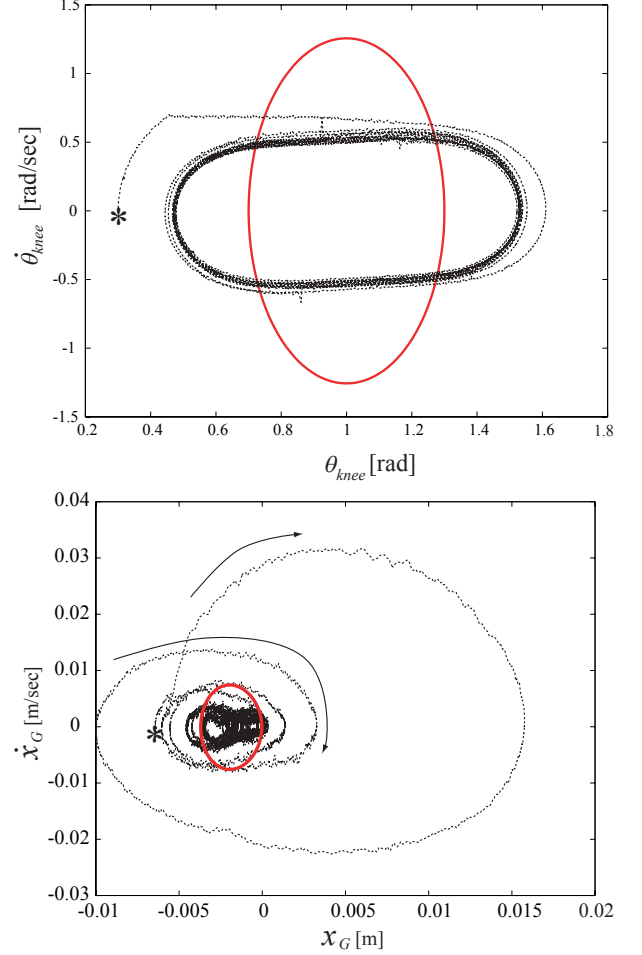


Fig. 12. Trajectories of the knee joint angle  $\theta_{knee}$  and  $x_G$

the dashed lines are the experimental data and the mark '\*' mean the initial values. Because each joints of the robot system is controlled by the local PD feedback controller, we assume the high gain feedback and input the reference angles. Besides the large friction of the geared motor, the model uncertainty is not small. This is because the motion trajectory is much different from the given trajectory  $\Xi$ . Figure 13 shows the motion of the hip joint that moves for the stabilization of the humanoid robot. Figure 14 shows the generated squat motion of the humanoid robot. The stable motion is realized.

#### V. CONCLUSIONS

In this paper, we develop the motion emergency system for humanoid robots using the attractor design of the non-linear dynamical system. In this method, the autonomous motion emergency is realized by designing a controller so that the dynamics of the environment, the robot body

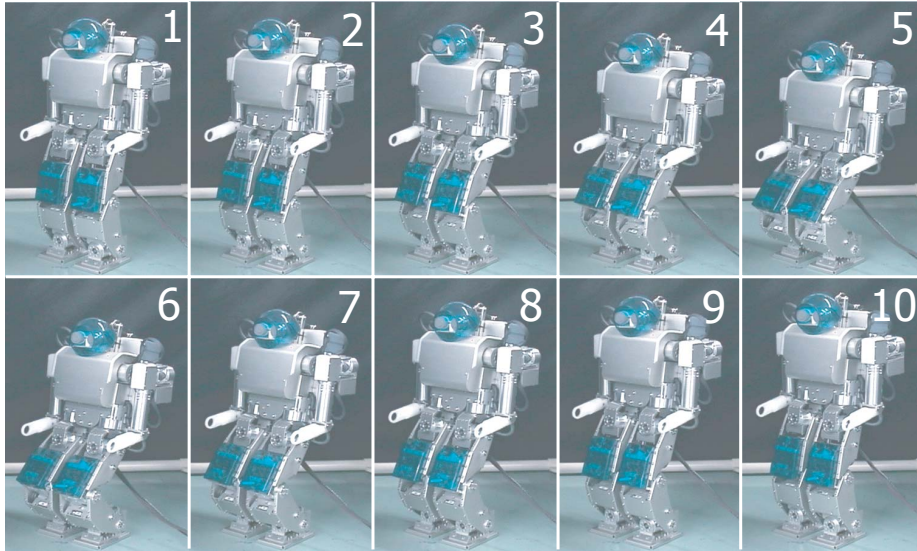


Fig. 14. Generated squat motion

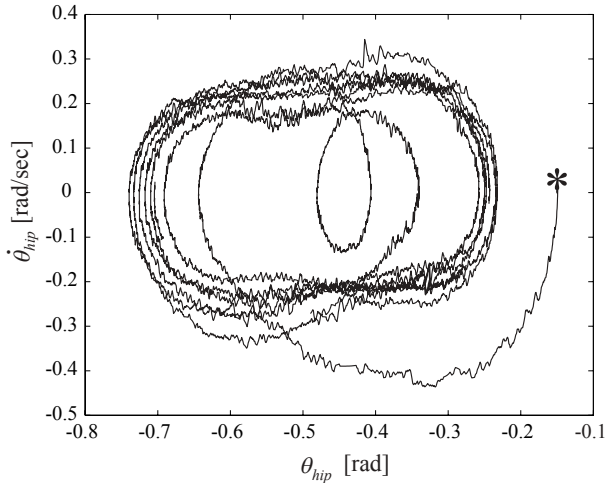


Fig. 13. Trajectories of the hip joint  $\theta_{hip}$

and the information processing system have an attractor on the motion trajectory through the interaction between each dynamics. The results of this paper are as follows.

- 1) We develop the design method of dynamics that has an attractor on the closed curved line considering the robot body dynamics.
- 2) The designed dynamics works as a controller by which the closed loop system has an equilibrium trajectory from the control engineering point of view.
- 3) The proposed method is based on the vector field design and function approximation. To show the controller design process, the simple numerical example is illustrated.
- 4) The motion emergency is realized using the humanoid robot in the real world, which evaluates the proposed method.

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