2009 IEEE International Conference on Robotics and Automation
Kobe International Conference Center
Kobe, Japan, May 12-17, 2009

Controller Decomposition and Combination Design of Body / Motion Elements based on Orbit Attractor

Masafumi Okada and Masaaki Watanabe
Dept. of Mechanical Sciences and Engineering, Tokyo TECH
2-12-1 Oookayama Meguro-ku Tokyo, 152-8552 JAPAN
okada@mep.titech.ac.jp, watanabe.m.aa@m.titech.ac.jp

***Abstract*— Robot control systems consist of a feedback controller and reference motion pattern. They are designed based on the robot dynamics and coupled with each other, and it requires lots of calculation to obtain them. So far, we have proposed controller design method based on orbit attractor of nonlinear dynamics. Because the controller yields one motion for one robot, we can assume that the controller includes information of motion and body elements. If those elements can be decomposed from the controller, a new controller can be easily designed by the combination of these elements. So in this paper, we propose the motion and body elements design method with Lagrange's method of undetermined multipliers based on robot dynamics, and combination design method of a new controller using these elements. The effectiveness of the proposed method is evaluated by experiments with tapping dance robots.**

I. INTRODUCTION

Robot control systems consist of a stabilizing controller K and reference motion pattern x^{ref} as shown in figure 1. Because K and x^{ref} are designed based on the specified

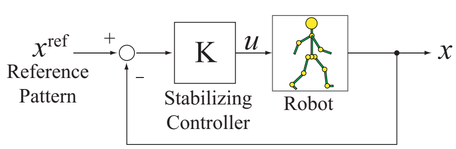

Fig. 1. Robot control system

robot dynamics, motion and environments, the robot requires the same number of control systems as motions and situations in which it moves. Moreover, it needs a large amount of calculation to design a controller and motion pattern for the robots in the real world, and another motion or controller requires more calculation to be re-designed. For example, in the case of biped robot, x^{ref} have to be designed under the physical law of walking dynamics (e.g. ZMP constraint) which depends on the changing environments. To reduce the calculation costs, a new controller design method using the existing controller will be effective.

On the other hand, Okada[1][2] proposed a controller design method to make the state variable of the robot entrain to a specified orbit in the state space. This method uses the functional approximation of the defined vector field in the state space, and designs both a stabilizing controller and a motion reference pattern simultaneously with low calculation cost. The same concept of the robot controller design is applied to cooperative task with human[3] and surgery robot[4] because of high robustness and flexibility for environments. Because the controller depends on the robot body dynamics and the specified motion, it contains both robot body and motion elements. By extracting the body and motion elements from the existing controllers and connecting these elements, a new controller will be obtained.

In this paper, we propose a controller decomposition and combination design method based on the existing controllers for a new controller design considering the physical relationship between robot dynamics, orbit and controller input. The motion element is a common element of the same motions for different robots, the body element is a common element of the same robot for various motions. By combining each element, a new controller for new robots or motions is designed. This concept is similar way to obtain a symbol of body and motion[5][6]. The effectiveness of the proposed method is evaluated by experiments using a tapping dance robot.

II. CONTROLLER DESIGN BASED ON ATTRACTOR OF NONLINEAR DYNAMICS

A. Attractor design

In reference [2], the robot motion emergence method based on orbit attractor is proposed. In this section, the controller design method is summarized.

Consider the robot body dynamics represented by the following difference equation in discrete time domain;

$$x[k+1] = f(x[k]) + g(x[k], u[k]) \tag{1}$$

where $x[k]$ is the state variable, $u[k]$ is the input of dynamics with time stamp k. The controller is designed by the nonlinear function of x as follow;

$$u[k] = h(x[k]) \tag{2}$$

so that $x[k]$ is entrained to a specified closed curved line Ξ;

$$\Xi = \left[\begin{array}{cccc} \xi_1 & \xi_2 & \cdots & \xi_N \end{array} \right] \tag{3}$$

in the state space. With the mathematical representation, the solution of the simultaneous equations (1) and (2) converges to Ξ at $k \rightarrow \infty$, which means the controller makes the attractor Ξ of the robot body dynamics.

The controller is designed by polynomial of ℓ-th order power of x as follow;

$$u[k] = \Theta\phi(x[k]) \tag{4}$$

where Θ is a coefficient matrix of polynomial and $\phi(x)$ expands the state vector x to the power vector of x. For example, $x \in \mathcal{R}^2$ and $\ell = 2$ cause ϕ as;

$$\phi(x) = \begin{bmatrix} 1 & x_1 & x_2 & x_1^2 & x_1 x_2 & x_2^2 \end{bmatrix}^T \tag{5}$$

$$x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \tag{6}$$

In this method, the motion is emerged by the interaction between robot body dynamics in equation (1) including environment and controller in equation (2). The explicit motion pattern x^{ref} is not required, but Θ contains implicit information of motion pattern and robot body because the controller causes a stable robot motion entraining the state variable.

B. Controller design method

Controller Θ is designed by functional approximation using many sets of realizable $(x,\ u)$. Equation (1) is approximated to linear form by Taylor expansion around $x = x[k]$, $u = u[k]$ as follows;

$$x[k+1] = A(x[k])x[k] + B(x[k])u[k] + C(x[k]) \tag{7}$$

$$A(x[k]) = \frac{\partial f(x[k])}{\partial x} + \frac{\partial g(x[k], u[k])}{\partial x} \tag{8}$$

$$B(x[k]) = \frac{\partial g(x[k], u[k])}{\partial u} \tag{9}$$

$$C(x[k]) = -\left(\frac{\partial f(x[k])}{\partial x} + \frac{\partial g(x[k], u[k])}{\partial x}\right)x[k] - \frac{\partial g(x[k], u[k])}{\partial u}u[k] \tag{10}$$

By using input sequence $\{u[a],\ u[a+1],\ \cdots u[a+b-1]\}$, the sequence of state vector $\{x[a],\ x[a+1],\ \cdots x[a+b]\}$ is represented by

$$\boldsymbol{X}_{a+1}^{a+b} = \boldsymbol{A}x[a] + \boldsymbol{B}\boldsymbol{U}_a^{a+b-1} + \boldsymbol{C} \tag{11}$$

$$\boldsymbol{X}_{a+1}^{a+b} = \begin{bmatrix} x[a+1]^T & \cdots & x[a+b]^T \end{bmatrix}^T \tag{12}$$

$$\boldsymbol{U}_a^{a+b-1} = \begin{bmatrix} u[a]^T & \cdots & u[a+b-1]^T \end{bmatrix}^T \tag{13}$$

$$\boldsymbol{A} = \begin{bmatrix} A(x[a])^T & \cdots & \left(\prod_{m=a}^{a+b-1} A(x[m])\right)^T \end{bmatrix}^T$$

$$\boldsymbol{B} = \begin{bmatrix} B(x[a]) & \cdots & 0 \\ \vdots & \ddots & \\ \left(\prod_{m=a+1}^{a+b-1} A(x[m])\right) B(x[a]) & \cdots & B(x[a+b-1]) \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} C(x[a]) \\ \vdots \\ C(x[a+b-1]) + \sum_{n=a}^{a+b-2}\left(\left(\prod_{m=k+1}^{a+b-1} A(x[m])\right) C(x[n])\right) \end{bmatrix}$$

For easy expression, subscripts and super-scripts which mean initial and final time number respectively are omitted in the following equations. From equation (11), the input sequence $\boldsymbol{U}$ that carries $x[a]$ along with ξ_k is obtained by

$$\boldsymbol{U} = \boldsymbol{B}^{\#}\left(\Xi - \boldsymbol{A}x[a] - \boldsymbol{C}\right) \tag{14}$$

$$\Xi = \begin{bmatrix} \xi_{a+1} & \xi_{a+2} & \cdots & \xi_{a+b} \end{bmatrix}^T \tag{15}$$

$$\boldsymbol{B}^{\#} = \left(\boldsymbol{B}^T\boldsymbol{B}\right)^{-1}\boldsymbol{B}^T \tag{16}$$

And the locus of state vector yielded by the corresponding input sequence is calculated based on equation (11), which give us the realizable sets of $(x,\ u)$. By setting many initial

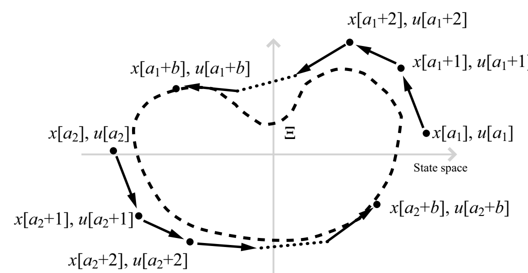

Fig. 2. Calculation to obtain the sets of realizable $(x,\ u)$

value $x[a]$ around Ξ as shown in figure 2, Θ is designed by functional approximation to minimize the following cost function $J_{\Theta_i^j}$;

$$\Theta_i^j = \arg\min_{\Theta_i^j} J_{\Theta_i^j} \tag{17}$$

$$J_{\Theta_i^j} = \sum_k \left\| u_i^j[k] - \Theta_i^j \phi(x_i^j[k]) \right\|_2 \tag{18}$$

where subscript i means the motion index ($i = 1,\ 2,\ \cdots$) and super-script j means the robot body index ($j = A,\ B,\ \cdots$).

III. Design of motion and body elements

A. Decomposition of controller

In this section, controller decomposition is illustrated. Consider equations (4) and (7). In the state space, these equations mean that the vector from $x[k]$ to $x[k+1]$ as;

$$\delta_k = x[k+1] - x[k] \tag{19}$$

is separated into the input component $\delta_k/_B$ and its perpendicular component $\delta_k/_{B^\perp}$ as shown in figure 3, where $[\ \cdot\ ]/_B$ means the projection of vector onto span(B). These components are defined by

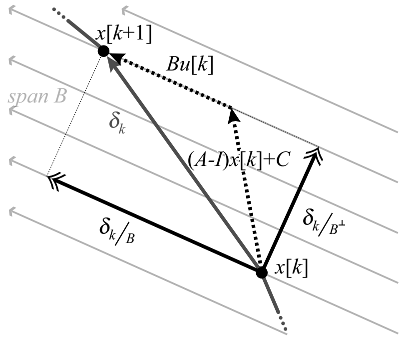

Fig. 3. Decomposition of $x[k+1]-x[k]$

$$\begin{aligned}\delta_k/_B &= \{(A(x[k])-I)x[k]+C(x[k])\}/_B \\ &\quad +B(x[k])u[k] \qquad (20)\\ \delta_k/_{B^\perp} &= \{(A(x[k])-I)x[k]+C(x[k])\}/_{B^\perp} \qquad (21)\end{aligned}$$

and they are calculated by

$$\begin{aligned}\delta_k/_B &= B(x[k])B(x[k])^{\#}(x[k+1]-x[k]) \qquad (22)\\ &= B(x[k])B(x[k])^{\#}\{(A(x[k])-I)x[k] \\ &\quad +C(x[k])\}+B(x[k])u[k] \qquad (23)\\ \delta_k/_{B^\perp} &= (I-B(x[k])B(x[k])^{\#})\{(A(x[k])-I)x[k] \\ &\quad +C(x[k])\} \qquad (24)\end{aligned}$$

Because $\delta_k/_{B^\perp}$ is uniquely defined by the robot body dynamics, it represents the robot body component, and the rest component $\delta_k/_B$ represents the motion elements.

On the other hand, let me consider the decomposition of the input as;

$$\begin{aligned}B(x[k])u[k] &= x[k+1]-A(x[k])x[k]-C(x[k]) \qquad (25)\\ &= B(x[k])(u_b[k]+u_m[k]) \qquad (26)\\ u_b[k] &= B(x[k])^{\#}\{-(A(x[k])-I)x[k]-C(x[k])\} \qquad (27)\\ u_m[k] &= B(x[k])^{\#}(x[k+1]-x[k]) \qquad (28)\end{aligned}$$

and u_b and u_m satisfy;

$$\begin{aligned}B(x[k])u_b[k] &= (A(x[k])-I)x[k]+C(x[k])-\delta_k/_{B^\perp} \qquad (29)\\ B(x[k])u_m[k] &= \delta_k/_B \qquad (30)\end{aligned}$$

Because Bu_b does not contain the information of $x[k+1]$, it consists of the body element, and Bu_m consists of the motion element. From these considerations, we can conclude that u_b

$$u_b[k] = B(x[k])^{\#}(A(x[k])-I)x[k]+C(x[k]) \qquad (31)$$

represents the body element and u_m

$$u_m[k] = B(x[k])^{\#}(x[k+1]-x[k]) \qquad (32)$$

represents the motion elements of the input. Here we assume that u_b and u_m are represented by the function of $x[k]$ as;

$$\begin{aligned}u_b[k] &= \Gamma\phi(x[k]) \qquad (33)\\ u_m[k] &= \Lambda\phi(x[k]) \qquad (34)\end{aligned}$$

using the polynomial of ℓ-th order power of $x[k]$, and the controller is represented by

$$\Theta\phi(x[k]) = (\Gamma+\Lambda)\phi(x[k]) \qquad (35)$$

where Γ is the body element and Λ is the motion element of the controller consisting of the coefficient matrix of polynomial of power of x, and the controller is decomposed as;

$$\Theta = \Gamma+\Lambda \qquad (36)$$

B. Controller decomposition for multi motions

Let us consider Θ_1, Θ_2 which generate motion 1 and 2 respectively for one robot. They are decomposed to Λ_1, Λ_2 and same Γ like equation (36). Because these controllers are for one robot, Γ is common, and the following constraint is satisfied.

$$\Theta_1-\Lambda_1 = \Theta_2-\Lambda_2 (=\Gamma) \qquad (37)$$

Figure 4 shows the difference of input to generate motion 1 and 2. For the same state value $x_1[k]=x_2[k]$, $x[k]$ advances

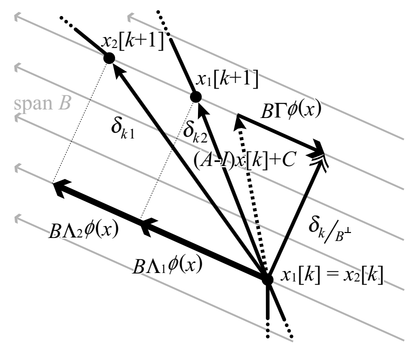

Fig. 4. Difference of motion depending on the controller input

$(A-I)x[k]+C$ according to the robot dynamics without input. The projection of $(A-I)x[k]+C$ onto span(B) is represented by

$$\{(A-I)x[k]+C\}/_B = -B\Gamma\phi(x[k]) \qquad (38)$$

which is common for both motions. The difference of motion is caused by the difference between $\delta_{k1}/_B$ and $\delta_{k2}/_B$

$$\begin{aligned}\delta_{k1}/_B &= B(x[k])\Lambda_1\phi(x[k]) \qquad (39)\\ \delta_{k2}/_B &= B(x[k])\Lambda_2\phi(x[k]) \qquad (40)\end{aligned}$$

which is the difference of the motion elements of the controllers.

C. Design method of motion and body element

Because Λ is represented by equations (32) and (34), Λ is obtained by minimizing the following cost function J_{Λ_i};

$$\Lambda_i = \arg\min_{\Lambda_i} J_{\Lambda_i} \qquad (41)$$

$$J_{\Lambda_i} = \sum_k \left\| \{B(x_i[k])\}^{\#} x_i[k+1] - \Lambda_i\phi(x_i[k]) \right\| \qquad (42)$$

using the obtained sets of $x[k]$ in section II-B. However, Θ_i and Λ_i satisfy the constraint

$$\Theta_i - \Lambda_i = \text{Common value } (= \Gamma) \tag{43}$$

the optimized solution using Lagrange's method of undetermined multipliers with constrain condition is required. The cost function to design Θ_i and Λ_i is written by J as

$$J = J_{\Theta_i^j} + J_{\Lambda_i} + \sum_{j,i\neq k,\ell} (\Theta_i^j - \Lambda_i - \Theta_k^j + \Lambda_k)\lambda_\ell^T \tag{44}$$

$$J_{\Lambda_i} = \sum_j \sum_k \left\| \left\{ B(x_i^j[k]) \right\}^{\#} x_i^j[k+1] - \Lambda_i \phi(x_i^j[k]) \right\| \tag{45}$$

where λ_ℓ are Lagrange's undetermined multipliers.

D. Controller design with the combination of motion and body elements

Once Θ_i and Λ_i are designed, new controllers are obtained by the combination of Γ^j and Λ_i. Figure 5 shows an example

	Robot α	Robot β
Motion 1	Θ_1^α : Γ^α Λ_1	Θ_1^β : Γ^β Λ_1
Motion 2	Θ_2^α : Γ^α Λ_2	Θ_2^β : Γ^β Λ_2
Motion 3	Θ_3^α : Γ^α Λ_3	Θ_3^β : $\boxed{\Gamma^\beta}$ $\boxed{\Lambda_3}$

Fig. 5. New controller design using Γ and Λ

of the new controller design. Using Λ_3 and Γ^β, a new controller is obtained by

$$u_3^\beta = (\Gamma^\beta + \Lambda_3)\phi(x) \tag{46}$$

$$= (\Theta_1^\beta - \Lambda_1 + \Lambda_3)\phi(x) \tag{47}$$

for motion 3 of robot β that corresponds to Θ_3^β.

IV. Combination Design of Controller for Tapping Dance Robot

A. Experimental setup

In this section experiments are performed to evaluate the proposed decomposition and combination design method of controllers using tapping dance robots shown in figure 6. There are two robots, one is robot α (the larger) and the other is robot β (the smaller). The robot configuration is in figure 7. In the configuration, the robots are controlled with torque control for dynamic motion. The robots make the tapping dance as shown in figure 8. By changing the grounding foot, the robot steps continuously. The state variable x consists of lower body rotational angle θ, lower body rotational velocity $\dot\theta$, head rotational angle ϕ and head rotational velocity $\dot\phi$ as follow.

$$x = \begin{bmatrix} \theta & \dot\theta & \phi & \dot\phi \end{bmatrix}^T \tag{48}$$

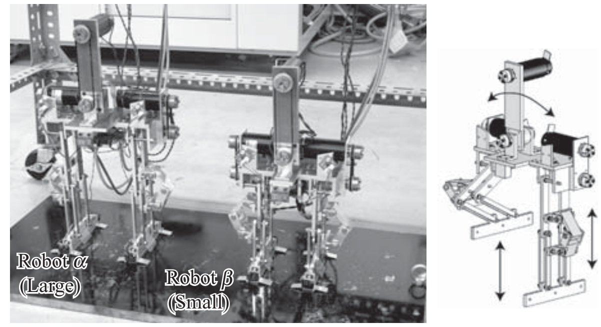

Fig. 6. Tapping dance robots

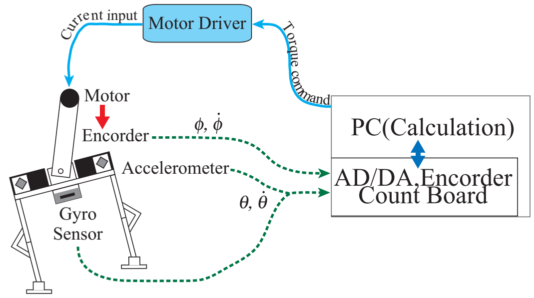

Fig. 7. Apparatus of the tapping dance robots

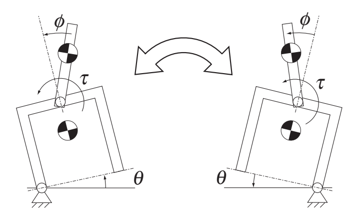

Fig. 8. Motion and modeling of tapping dance robots

They are contiguous values in spite of the change of grounding foot. And we assume that the foot impacts to ground as completely inelastic collision. Detail on the equations of motion of the tapping dance robots are written in [2]. The difference of the motion is evaluated by the difference of the frequency of tapping dance.

B. Design of Θ and Λ

We design controllers $\Theta_i^\alpha (i = 1 \sim 3)$ and $\Theta_1^\beta, \Theta_2^\beta$ for motion $1 \sim 3$ of robot α and β. Table.I shows the motion number and its target frequency. Note that the frequency of the motions are calculated by the average of motion cycles, because the tapping dance motion dose not have constant frequency because of the effect of external force such as an impact force of changing legs.

In the first experiments, the proposed method is evaluated. The controllers Θ_1^α, Θ_2^α, Θ_1^β, Θ_2^β and motion elements Λ_1, Λ_2 are designed. Figure 9 show the result of motion 2 for robot β using combination of Θ_1^β and motion elements Λ_1, Λ_2. The input is calculated by

$$\widehat{u}_2^\beta[k] = (\Theta_1^\beta - \Lambda_1 + \Lambda_2)\phi(x[k]) \tag{49}$$

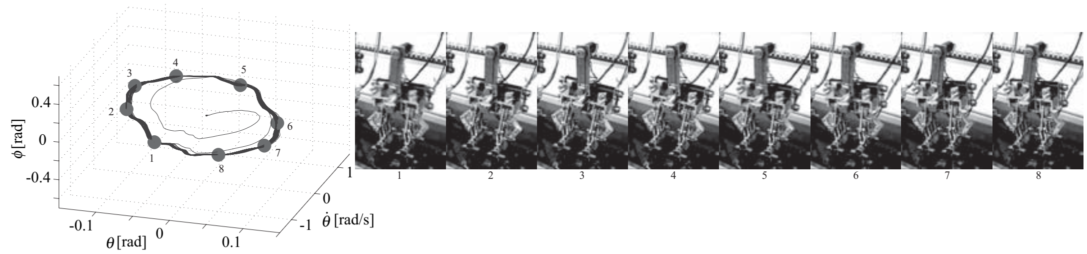

Fig. 9. Experiment with $\Theta_1^\beta - \Lambda_1 + \Lambda_2$.

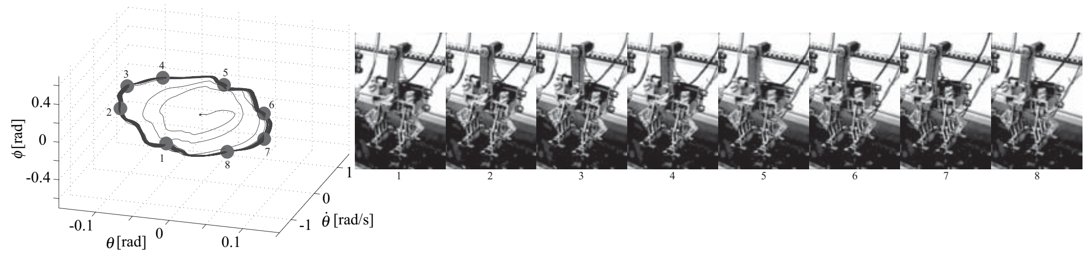

Fig. 10. Experiment with Θ_2^β.

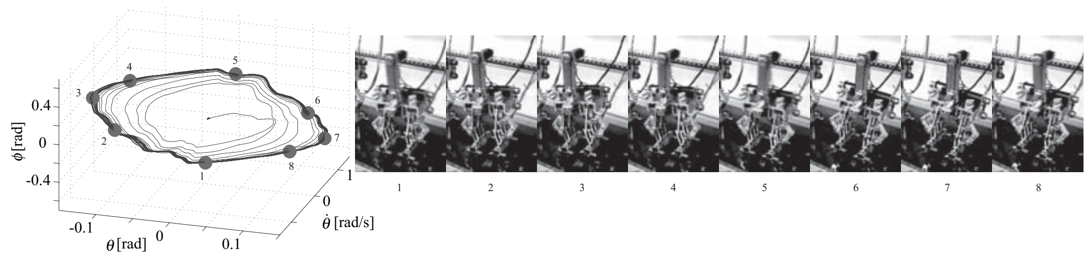

Fig. 11. New motion emergence with $\Theta_1^\beta - \Lambda_1 + \Lambda_3$.

TABLE I

ROBOT MOTION AND ITS FREQUENCY

Robot	Motion number	Target frequency	Actual frequency
α	1	2Hz	1.9Hz
	2	1.5Hz	1.5Hz
	3	1Hz	0.95Hz
β	1	2Hz	1.9Hz
	2	1.5Hz	1.5Hz

The state space is four dimensional space, however, only θ, $\dot{\theta}$ and ϕ space is shown. The solid line shows the orbit in the state space and some robot pictures of each point are illustrated around the orbit. It can see that the motion is entrained to a specified orbit.

For comparison, the tapping dance of robot β and motion 2 using the controller Θ_2^β

$$u_2^\beta[k] = \Theta_2^\beta \phi(x[k]) \tag{50}$$

is shown in figure 10. Because the results of figure 9 and 10 are same, it indicates the decomposition and combination design of the controller is effective. Frequency of these motions is 1.88Hz.

C. New controller design with combination of elements

The new controller for robot β and motion 3 is designed based on the elements Γ^β and Λ_3 as

$$\widehat{u}_3^\beta = (\Theta_1^\beta - \Lambda_1 + \Lambda_3)\phi(x) \tag{51}$$

By using $\widehat{u}_3^\beta$, the tapping dance is realized. The result is shown in figure 11. Robot β is entrained to the tapping dance with its frequency about 1.15Hz.

And we verified the proposed method about not only simple motion $1 \sim 3$, but also complex motion 4. Figure 12 represents the grounding time of the each foot while the tapping dance robot α is moving. Motion $1 \sim 3$ have constant rhythms, on the other hand motion 4 has a changing rhythm of the tapping dance. Figure 13 represents a x orbit of the motion 4 with robot α. Here we remark that the x orbit in the figure 13 dose not cross in the 4 dimensional state space. Using the proposed method, motion element Λ_4 is designed using Θ_4^α just as Λ_3 already is done. Based on the existing controller Θ_1^β and motion elements Λ_1, Λ_4, a new controller for the robot β is designed as

$$\widehat{u}_4^\beta = (\Theta_2^\beta - \Lambda_2 + \Lambda_4)\phi(x) \tag{52}$$

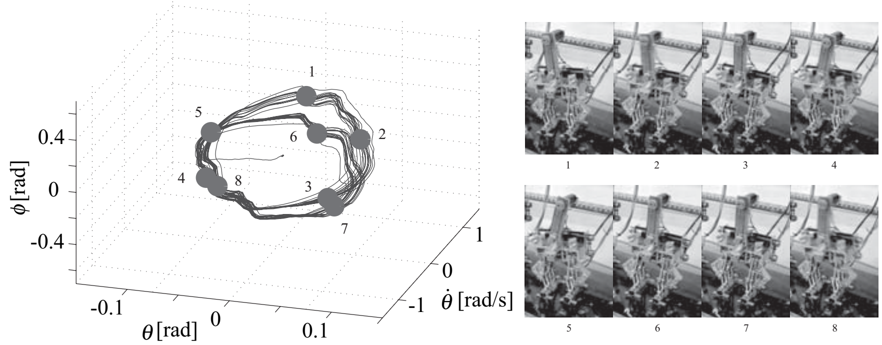

Fig. 14. New motion emergence usign $\Theta_1^{\beta} - \Lambda_1 + \Lambda_4$.

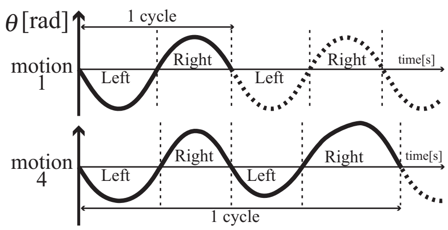

Fig. 12. Step comparison between motion1 and motion4 of tapping dance robot.

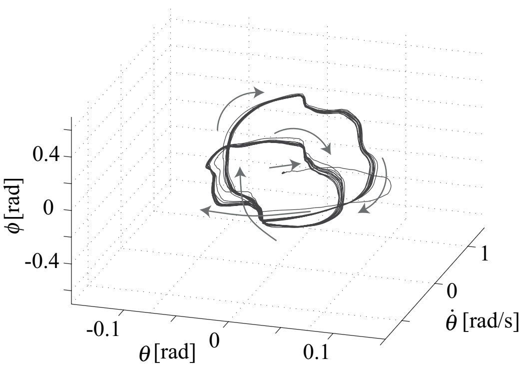

Fig. 13. Tapping dance emergence using Θ_4^{α} for Robotα.

Experimental result using equation (52) of robot β is shown in figure 14. In the result, the proposed method is effective about the motion 4 with comlex orbit by controller design with combination of motion and body elements. This result shows that combination of motion and robot body elements is effective for new controller design.

V. Conclusions

In this paper, we proposed the decomposition and combination design method of the controller based on orbit attractor. The results of this paper are as follows;

1) The decomposition method of controller into motion and body elements is proposed. In this method, the appropriate decomposition is discussed based on the robot body dynamics in the state space.
2) The constraint of the decomposition is discussed and calculation method is proposed using Lagrange's method of undetermined multipliers.
3) Based on the decomposed elements, a new controller design method is proposed which is a simple way with the sum of motion and body elements.
4) The proposed methods are evaluated by the experiments of the complex motion with the robots.

Acknowledgment

This research is supported by the "Motion emergence and mutual progression of robot body and intelligence from the dynamical point of view" under Grants-in-Aid for Scientific Research (Category Encouragement of Young Scientists (A)) of Japan Society for the Promotion of Science.

References

[1] M. Okada, K. Osato, and Y. Nakamura, "Motion emergence of humanoid robots by an attractor design of a nonlinear dynamics," in *Proc. of the IEEE International Conference on Robotics and Automation (ICRA'05)*, 2005, pp. 18–23.

[2] M. Okada and K. Murakami, "Robot communication principal by motion synchronization using orbit attractor," in *Proc. of the IEEE International Conference on Robotics and Automation (ICRA'07)*, 2007, pp. 2564–2569.

[3] T. Sato, M. Hashimoto, and M. Tsukahara, "Synchronization based control using online design of dynamics and its application to human-robot interaction," in *Proc. of the 2007 IEEE International Conference on Robotics and Biomimetics*, 2007, pp. 652–657.

[4] H. Mayer, I. Nagy, A. Knoll, E. U. Braun, R. Bauernschmitt, and R. Lange, "Adaptive control for human-robot skilltransfer : Trajectory planning based on fluid dynamics," in *Proc. of the IEEE International Conference on Robotics and Automation (ICRA'07)*, 2007, pp. 1800–1807.

[5] T. Ogata, S. Sugano, and J. Tani, "Acquisition of motion primitives of robot in human-navigation task: Towards human-robot interaction based on Quasi-Symbol," *Journal of Japanese Society of Artificial Intelligence*, vol. 20, no. 3, pp. 188–196, 2005.

[6] T. Inamura, H. Tanie, I. Toshima, and Y. Nakamura, "An approach from motion generation/recognition to intelligence based on mimesis principle," in *Proc. of the 2nd International Symposium on Adaptive Motion of Animals and Machines (AMAM2003)*, 2003, pp. SaA–II–1(CD–ROM).