ATTRACTOR DESIGN OF DYNAMICS BASED ON ENERGY DISTANCE IN STATE-SPACE FOR LINEAR SYSTEM

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Abstract: Robot motions are generated based on stabilizing controllers and reference motion patterns. On the other hand, human motions are determined through the interaction between the body and its environments. Motion patterns are not prepared a priori but emerge as the results of an entrainment phenomenon of the dynamics. So far, we have developed a controller design method that makes a dynamics entrain to the specific closed curved line. However, the obtained attractor is sometimes different from a desired one. That is fatal error for a robot motion with a drastic change of the dynamical characteristic of the robot body through the motion. In this paper, we develop a modified attractor design method based on energy distance in the state space.

1 INTRODUCTION

For industrial robots, the robot control systems have been designed using reference motion patterns and stabilizing controllers as shown in figure 1. The

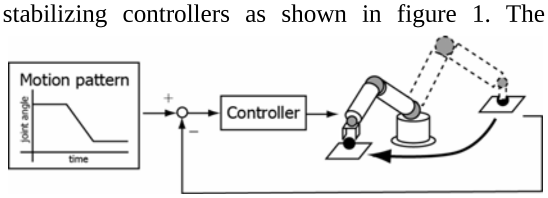

Figure 1: Motion control system for industrial robots

reference motion patterns are designed considering environments where the robot works and controllers are designed based on the robot body dynamics. For industrial robots, because the environments are fixed and the purpose of the control system is a precise task execution, this type of control system is suitable and many effective results have been reported. The main purpose of the controller is making the robot trajectory coincide to the reference motion pattern and ensures the robust stability of the closed loop system. The same strategy is employed for humanoid robots. The reference motion patterns are designed so that the robot dynamics (zero moment point : ZMP and center of gravity : COG) satisfies the dynamical constraints with environments. However, because humanoid robots move in unknown environments, the fixed motion patterns are not appropriate and the robustness of the controller is not sufficient.

On the other hand, the human motions are generated through the interaction between body dynamics, information processing and environments as shown in figure 2. The motion patterns are not prepared a priori but emerged as the results of the interaction of these dynamics. This concept corresponds with "embodiment" [Pfeifer, 2001] that represents a close relationship between body and intelligence in the brain science research field. For intelligent robots, a new control method that defines the motion pattern autonomously by the interaction with environments in real-time is required in the changing environments.

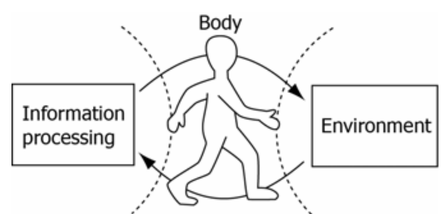

Figure 2: Motion generation of the human

From mathematics and control engineering points of view, the motion generation through the interaction between the body and environments, interpreted into the entrainment phenomenon of the nonlinear dynamics, and the generated motion pattern corresponds to an orbit attractor. The robot body dynamics is controlled so that it is entrained to a specific closed curved line from any initial points, which yields the motion and motion pattern. Some researchers have challenged to design a motion generation system based on dynamics approach. Kotosaka [Kotosaka, 2000], Tsujita [Tsujita, 2003] and Ijspeert [Ijspeert, 2002] proposed the motion generation system using central pattern generator (CPG). Sekiguchi [Sekiguchi, 2000] proposed a chaotic dynamics based method for a mobile robot. However these methods use existing dynamics, the parameter adjustment with trial and error plays an important role for designing entrainment phenomenon, which means the systematic design algorithm does not exist. Tani [Tani, 2003] proposed recurrent neural network based attractor design method. Because the existence of orbit attractor is not guaranteed, this method stays in analysis of phenomenon. Kawashima [Kawashima, 2002] proposed linear dynamics design method to recognize the similarity of the pattern from signal processing point of view. Adachi [Adachi, 2003] designed dynamics for motion pattern generation with lyapunov function based method. These methods do not contain the robot body dynamics, and in [Adachi, 2003], it is difficult to design a dynamics in more than four dimensional space because of the complexity of the design algorithm.

On the other hand, we have proposed an attractor design method of nonlinear dynamics that leads motion emergence of the humanoid robot [Okada, 2005]. In this method, we set a desirable motion trajectory and design a controller so that this trajectory will be an orbit attractor with a polynomial representation in the state space of the robot body dynamics [Okada, 2002]. The bending knee motion of the humanoid robot is realized, however the controller is calculated using a least square method that minimizes the input power using multi-step ahead prediction of the state variable, a contradiction is caused in causality between input and trajectory generation. That yields a large difference between the desired trajectory and generated motion, which is a fatal error when the robot body dynamics is drastically changed while its motion.

In this paper, we declare the problem of the conventional method and propose modified methods based on energy distance in the state space. The proposed methods are applied to an inverted pendulum system, and the effectiveness of the proposed methods is illustrated.

2. ATTRACTOR DESIGN METHOD

2.1 Minimization of the input power

In this section, I will illustrate the attractor design method in reference [Okada, 2002]. For simplicity, the dynamics is assumed to be controllable and represented by the following linear discrete time difference equation,

$$x[k+1] = Ax[k] + Bu[k] \tag{1}$$

where $x[k] \in R^n$ is a state variable and $u[k] \in R^m$ is an input signal. In the design algorithm, $u[k]$ is represented by the function of $x[k]$ such that

$$u[k] = f(x[k]) \tag{2}$$

and the controller is designed so that the closed loop system

$$x[k+1] = Ax[k] + Bf(x[k]) \tag{3}$$

has an attractor on the specific closed curved line Ξ

$$\Xi = \begin{bmatrix} \xi_1 & \xi_2 & \cdots & \xi_N \end{bmatrix} \tag{4}$$

which is given. The controller is designed by defining a vector field (pairs of ($x[k]$, $u[k]$)) and approximating $f(x[k])$ by *l*-th order polynomial of $x[k]$.

It is assumed that Ξ is realizable, which means the sequence of input signal $u[k]$ (k = 1, 2, …) that moves the state variable along with Ξ exists. The controller is designed as follows.

[Step 1] Set x_i in *x*-space and find ξ_i that is nearest to x_i.

[Step 2] x_{i+j} is *j*-step ahead prediction of x_i that is represented by the following equation.

$$x_{i+j} = A^j x_i + \Gamma U \tag{5}$$
$$\Gamma = \begin{bmatrix} B & AB & \cdots & A^{j-1}B \end{bmatrix} \tag{6}$$
$$U = \begin{bmatrix} u_i^T & u_{i+1}^T & \cdots & u_{i+j-1}^T \end{bmatrix} \tag{7}$$

[Step 3] Because Γ is an extended controllable matrix with column full rank when \$ $j \geq n$, U that takes x_{i+j} onto ξ_{i+j} exists and obtained by

$$U = \Gamma^{\#}(\xi_{i+j} - A^j x_i) \tag{8}$$

[Step 4] By using the obtained U, we calculate x_k, (k = i+1, … , i+j-1) and obtain pairs of (x, u). By defining many initial points x_i and obtaining many pairs of (x, u), $f(x[k])$ in equation (2) is obtained by polynomial approximation.

2.2 Contradiction in causality

Though x_{i+j} is guaranteed to coincide to ξ_{i+j}, the trajectory from x_i to x_{i+j} is not specified a priori. Equation (8) means the minimization of the input power, and x_{i+1} , x_{i+2} , …, x_{i+j-1} is defined subsequently, which does not guarantee for the state variable to pass near Ξ. As shown in figure 3, when the motion of the dynamics is slow, the conventional method is effective (which means $x_k \approx \xi_k$, ($k = i+1, \ldots, i+j-1$)), however when the motion is fast, the obtained trajectory makes a short cut and can be distant from Ξ.

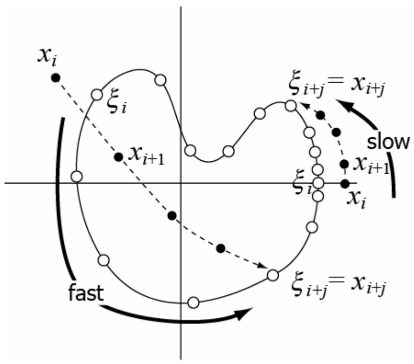

Figure 3: Desired closed curved line and obtained trajectory

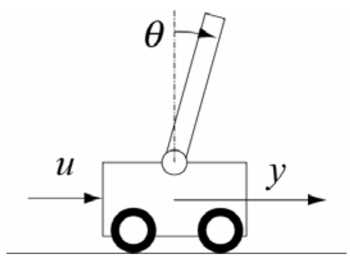

Figure 4: Inverted pendulum system

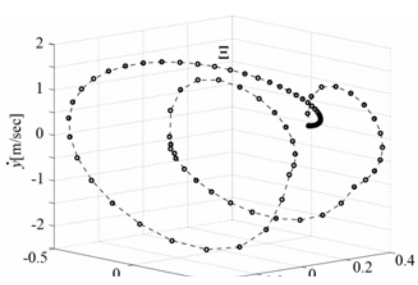

Figure 5: Reference motion pattern of the inverted pendulum system in state space

The following results show an example. Consider the inverted pendulum system as shown in figure 4. Setting θ (the rotational angle of the pendulum), y (position of the cart) and u (input force), and defining the state vector x as follows,

$$x = \begin{bmatrix} \theta & \dot{\theta} & y & \dot{y} \end{bmatrix}^T \tag{9}$$

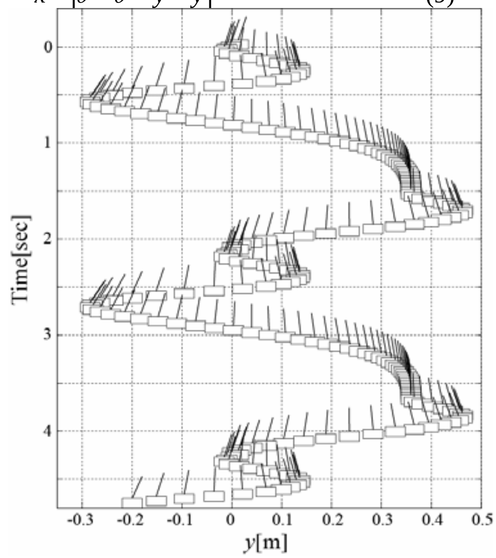

Figure 6: Reference motion pattern of the inverted pendulum system in the real world

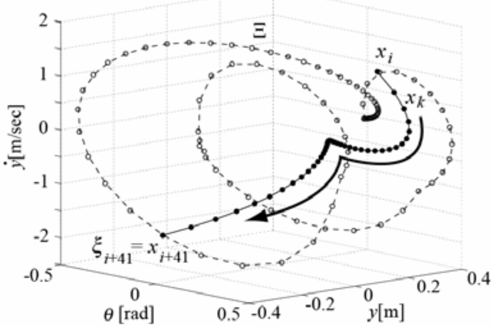

Figure 7: Trajectory of obtained x via conventional method

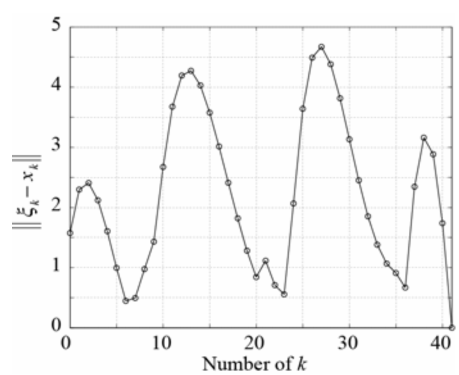

Figure 8: Norm of $\|\xi_k - x_k\|$ via conventional method

the dynamic equation is obtained. By discretizing the dynamics in a sampling time T, we obtain a discrete time dynamics in equation (1). We set Ξ as shown in figure 5 and the motion of the inverted pendulum in the real world is shown in figure 6.

Though x is a 4 dimensional vector, 3 dimensional space whose coordinates are θ, y and $\dot{y}$ are shown for simplicity. Based on Ξ, a controller is designed. Figure 7 shows Ξ and one example of the obtained trajectory x_k, ($k = i, \ldots, i+41$). Though x_{i+41} coincides to ξ_{i+41}, the trajectory makes short cut. Figure 8

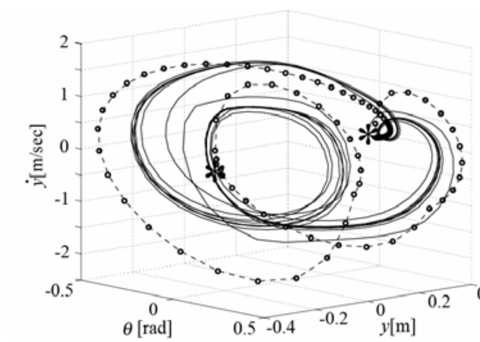

Figure 9: Trajectory of the dynamics via conventional method

shows $\|\xi_k - x_k\|$ using obtained x_k. $\|\xi_k - x_k\|$ does not decrease, which means x_k does not go along with ξ_k. From the obtained pairs of (x, u), $f(x[k])$ is approximated by 6th-order polynomial. Figure 9 shows the trajectory of the controlled dynamics

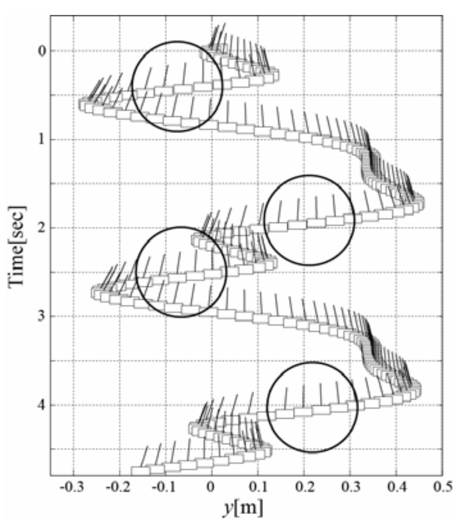

Figure 10: Motion of the inverted pendulum system in the real world with conventional method

starting from two initial points represented by "*". Though the dynamics is stabilized to one attractor, it is different from Ξ. The motion of the inverted pendulum system in the real world is shown in figure 10. The initial state variable is set to ξ_1. Comparing with figure 6, the velocity is sometimes small which is indicated by circles in figure 10.

In this example, because the control object is a linear system, the difference between the obtained trajectory and Ξ is not fatal error. When the system is nonlinear, we use the linear approximated system A_k, B_k around each \$$\xi_k$. Γ is set assuming x_k goes along with ξ_k, which means the difference between the obtained trajectory and Ξ is fatal. This problem is led by the contradiction in causality related to the input signal and trajectory, which causes unstable motion when the dynamic characteristic drastically changes through the motion such as walk motion. In the following section, a modified attractor design method is proposed.

3. TRAJECTORY-BASED ATTRACTOR DESIGN METHOD

3.1 Euclid distance-based design method

In this section, I will show an attractor design method minimizing the distance between x_k and ξ_k. From equation (1), we obtain the following equations.

$$X_{k+1} = \boldsymbol{A}x[k] + \boldsymbol{B}U \tag{10}$$

$$X_{k+1} = \begin{bmatrix} x[k+1] \\ x[k+2] \\ \vdots \\ x[k+j] \end{bmatrix}, \boldsymbol{A} = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^j \end{bmatrix} \tag{11}$$

$$\boldsymbol{B} = \begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{j-1}B & A^{j-2}B & \cdots & B \end{bmatrix} \tag{12}$$

Using these equations, we obtain U as follows.

$$U = \boldsymbol{B}^{\#}(\Xi_{k+1} - \boldsymbol{A}x_i) \tag{13}$$

$$\Xi_{k+1} = [\xi_{i+1}^T \quad \xi_{i+2}^T \quad \cdots \quad \xi_{i+j}^T]^T \tag{14}$$

Equation (13) means the minimization of the following criterion function J,

$$J = \sum_{\kappa=1}^{j} \|\xi_{i+\kappa} - x_{i+\kappa}\| \tag{15}$$

which corresponds to square summation of Euclid distance between x_k and ξ_k ($k = i+1,\ldots, i+j$) as sown

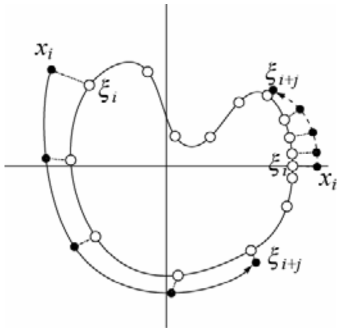

Figure 11: Definition of Euclid distance in the criterion function

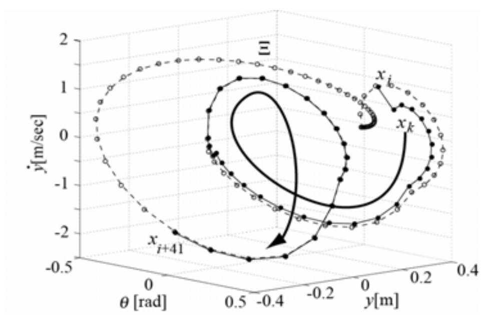

Figure 12: Trajectory of obtained x via the least square method

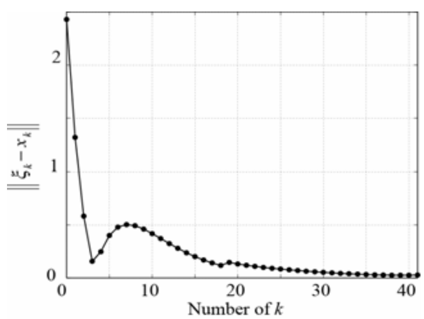

Figure 13: Norm of $\|\xi_k - x_k\|$ via the least square method

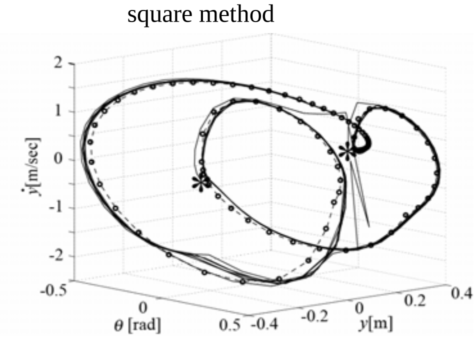

Figure 14: Trajectory of the dynamics via the least square method

in figure 11. By the same way as conventional method, we obtain some pairs of (x, u) and design a controller. Figure 12 shows Ξ and one example of x_k, ($k = i, \ldots, i$+41) like figure 7. Comparing to figure 7, x_k goes along with ξ_k. Figure 13 shows the obtained euclid norm $\|\xi_k - x_k\|$. x_k approaches ξ_k with increasing of k. Figure 14 shows the trajectory of the controlled dynamics. The controller is designed using 6th order polynomial which is same as previous section. Comparing with figure 9, the obtained trajectory is similar to Ξ.

3.2 Energy distance-based design method

In the Euclid distance-based method, the dynamics does not converge enough to the attractor. $x[k]$ takes other routes in each cycle and the trajectory draws thick line in figure 14. This is because the convergence rate of $\|\xi_k - x_k\|$ in figure 13 is low. In the following, I propose the modifying method.

When a state variable $x[k]$ of a stable dynamics converges to zero, Euclid distance $\| x[k] \|$ does not correspond to how the state vector $x[k]$ approaches to zero. Figure 15 shows a concept chart. The left hand

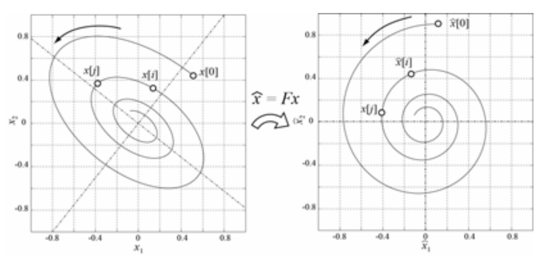

Figure 15: Convergence of dynamics

side shows the trajectory of $x[k]$ starting from $x[0]$ in state space. $\|x[j]\| < \|x[i]\|$ ($j > i$) is not always satisfied, which means $x[j]$ is more convergent than $x[i]$ but $\|x[j]\|$ can be likely larger than $\|x[i]\|$. On the other hand, as shown in the right hand side, x -coordinates is transformed to $\tilde{x}$ -coordinates by the transform matrix F and when $\| \tilde{x}[j] \| < \| \tilde{x}[i] \|$ ($j > i$) is always satisfied, Euclid distance $\| \tilde{x} \|$ corresponds to the convergence rate of the state variable in $\tilde{x}$ state space. In the following, calculation method of F is illustrated.

Consider a conservative system represented by the following differential equation.

$$\dot{x} = Ax \ , \quad x \in R^n \tag{16}$$

where we assume that A is diagonalizable. The state variable x starting from x_0 moves on the shell of a ellipsoid in n-dimensional space with its center on the origin. Let's obtain a matrix F that transforms the ellipsoid to sphere. Consider the eigen value decomposition of A as follows.

$$A = T\Lambda T^{-1} \tag{17}$$
$$\Lambda = \mathrm{diag}\{\lambda_1, \quad \lambda_2, \quad \cdots \quad \lambda_n\} \tag{18}$$

Because the dynamics is a conservative system,

$$\mathrm{Real}\,(\lambda(A)) = 0 \tag{19}$$

is satisfied. Here, we consider $\hat{x}$ defined by the following transformation.

$$\tilde{x} = T^{-1}x \tag{20}$$

The solution $\tilde{x}(t)$ of the differential equation (16) is represented by the following equation

$$\tilde{x} = \exp(\Lambda t)\tilde{x}_0 = \mathrm{diag}\{e^{\lambda_1 t}, \quad e^{\lambda_2 t}, \quad \cdots \quad e^{\lambda_n t}\}\tilde{x}_0 \tag{21}$$

where $\tilde{x}_0$ means the initial value. The inner product of $\tilde{x}(t)$ satisfies the following equation.

$$\tilde{x}^*(t)\tilde{x}(t) = x^T(t)\left(T^{-1}\right)^* T^{-1}x(t) \tag{22}$$
$$= \tilde{x}_0^* \,\mathrm{diag}\left\{e^{(\lambda_1^*+\lambda_1)t}, \quad \cdots \quad e^{(\lambda_n^*+\lambda_n)t}\right\}\tilde{x}_0 \tag{23}$$
$$= \tilde{x}_0^*\tilde{x}_0 = \text{Const.} \tag{24}$$

Though equation (24) represents a sphere, $\tilde{x}(t)$ -space consists of complex number. The singular value decomposition

$$\left(T^{-1}\right)^* T^{-1} = USU^T \tag{25}$$

gives the transformation matrix F with real number

$$\hat{x} = Fx \tag{26}$$
$$F = S^{1/2}U^T \tag{27}$$

that transforms the ellipsoid in x-space to a sphere in $\hat{x}$ -space. Equation (24) means Hamiltonian (conservation value) that corresponds to the energy of the system.

From these considerations, the attractor design method is modified as follows using F.

(1) Using defined x_i, find ξ_i that minimizes $\|F(\xi_i - x_i)\|$.

(2) Define j in equation (10) that satisfies $\|F(\xi_{i+j} - x_{i+j})\| < \Delta$, where Δ is a design parameter. j is calculated by

$$j = \frac{\log\Delta - \log\left(\|F(\xi_i - x_i)\|\right)}{\log\delta} \tag{28}$$

$$\delta^j \|F(\xi_i - x_i)\| < \Delta \tag{29}$$

where δ defines the convergence velocity of $x(t)$.

(3) By using a weighted least square method based of the following evaluation function,

$$J = \sum_{k=1}^{j} \delta^{-k} \|F(\xi_i - x_i)\| \tag{30}$$

substituting for equation (15), energy distance is evaluated.

Using the modified design method, we design a controller for the inverted pendulum system. Figure 16 shows the obtained x_k, ($k = i, \ldots, i+45$), figure 17 shows the value of $\|F(\xi_k - x_k)\|$. x_k converges to ξ_k in the sense of energy distance while k increases.

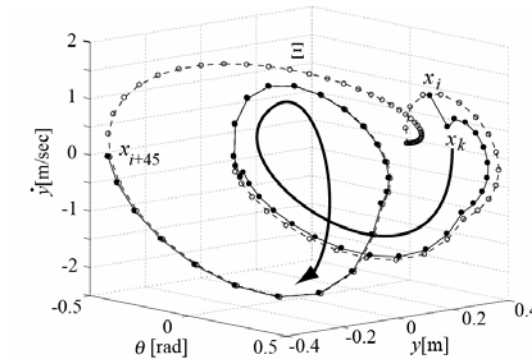

Figure 16: Trajectory of obtained x via energy distance method

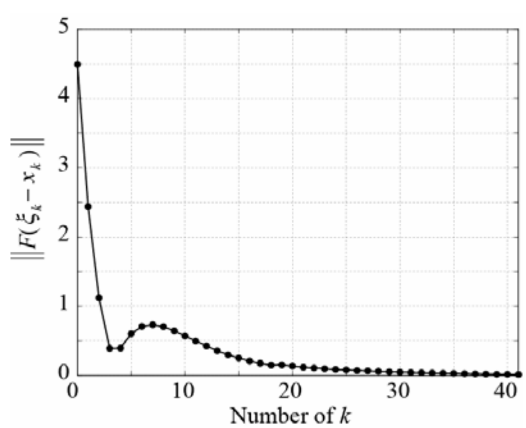

Figure 17: Norm of $\|F(\xi_k - x_k)\|$ via energy distance method

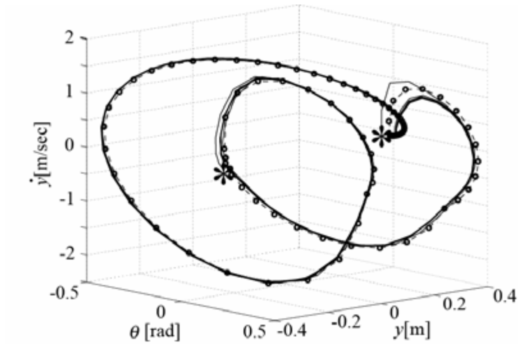

Figure 18: Trajectory of the dynamics via energy distance method

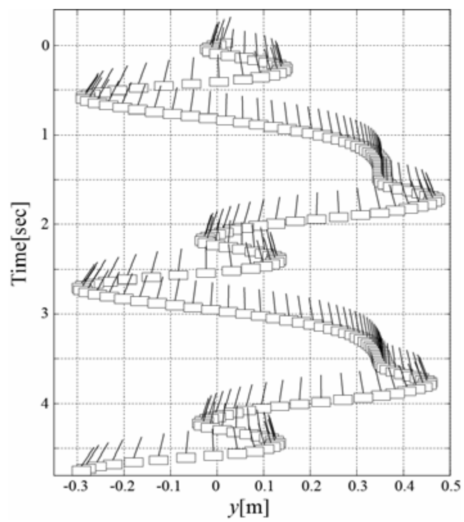

Figure 19: Motion of the inverted pendulum system via energy distance method

Figure 18 shows the designed attractor and figure 19 shows the motion of the inverted pendulum system in the real world. Comparing to figure 14, $x[k]$ converges Ξ and the trajectory is represented by a thin line.

4. CONCLUSIONS

In this paper, I proposed the modified attractor design method from the linear control engineering point of view.

(1) I declare the problem of the conventional method (causality of the relationship between the route and the input) that is the minimization of the input power.
(2) To overcome the problem, I propose the modified methods that minimize Euclidean distance of multi-steps ahead predictions and the desired trajectory.
(3) Moreover, we propose a new approach that evaluates the energy distance of the state variable, and shows that the designed attractor approaches the desired trajectory.

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BIOGRAPHY

Masafumi OKADA received his PhD from Kyoto University in 1996. He was a research associate of The University of Tokyo in 1997, he was a Lecture of The University of Tokyo in 2000, and he is now a Associate Professor of Tokyo Tech from 2004. His research interest is the simultaneous design of the robot mechanism and intelligent control using nonlinear dynamics point of view.