

# Optimal Filtering for Humanoid Robot State Estimators

Lekskulchai PONGSAK<sup>\*1</sup>, Masafumi OKADA<sup>\*1</sup>, Yoshihiko NAKAMURA<sup>\*1\*2</sup>

<sup>\*1</sup>Department of Mechano-Informatics, The University of Tokyo  
7-3-1 Hongo Bunkyo-ku, Tokyo 113-8656 Japan

<sup>\*2</sup>CREST Program, Japan Science and Technology Corporation  
email: hong@ynl.t.u-tokyo.ac.jp

**Abstract**— In controlling a humanoid robot, state estimators are used to estimate condition of motion and orientation of the body part which will be feedbacked to control systems to regulate level of the body along with controlling trajectory of center of gravity (C.G.) of the robot. In this research, an algorithm for state estimators is developed by applying the  $H_2$ -norm optimal filtering technique to a linearized dynamic model of the body part of the robot.

**Key Words:** Dynamic system filtering, State estimator, Humanoid robot

## 1. Introduction

In controlling a humanoid robot, the body should be regulated to be horizontal all times while its C.G. is being controlled along a planned trajectory. For this purpose, there must be a state estimator for estimating states of motion of the C.G. and orientation of the body part to be feedbacked to control systems.

Estimating states of the body part can be analyzed by treating it as a rigid body. Up till now, there has been much research concerning state estimation of rigid bodies. Most of them are used in the field of vehicle navigation where position and/or orientation of vehicles are determined. Generally, angular rate sensors incorporating with accelerometers are used and sensor data is fused together by using the Kalman filter or complementary filters<sup>1, 2, 3, 4)</sup>. In most research, only kinematic equations of rigid bodies are used in developing filters due to unknown input forces and moments. For example, in head tracking systems<sup>2)</sup>, the forces and moments exerted by muscles can not be determined. However, not using the equations of motion results in missing information about inertia of the objects. To obtain more accurate estimation, the equations of motion of the object being tracked should be included in developing filters such as in [4] where input forces and moments of an aircraft can be calculated from the position of aileron.

In case of humanoid robot systems, total forces and moments on the body part can be determined by using whole body dynamic equations of the robot because information of joint angles, forces and moments on both feet are available from encoders and force-moment sensors respectively. However, the whole body equations are very complex and require long computational time hence not practical to be solved in real-time.

In this paper, we develop a state estimator by using both kinematic equations and equations of motion of rigid bodies without trying to calculate input forces and moments. Instead, we design it as a filter that estimates states from sensor data in the presence of

external disturbance. In this case, the Kalman filter would not be appropriate because it requires the assumption that noises are white with zero means which is too restrictive. In order to design a filter that is more robust to noise characteristics, we approach by the  $H_2$ -norm optimal filtering technique.

## 2. Linearized Dynamic Equations

The dynamic equations of the body part and measurement equations can be written as follows<sup>5)</sup>

$$\left. \begin{aligned} m\dot{\mathbf{v}}^b + \boldsymbol{\omega}^b \times m\mathbf{v}^b &= \mathbf{f}^b \\ \mathbf{J}\dot{\boldsymbol{\omega}}^b + \boldsymbol{\omega}^b \times \mathbf{J}\boldsymbol{\omega}^b &= \boldsymbol{\tau}^b \end{aligned} \right\} \quad (1)$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \frac{\sin \phi}{\cos \theta} & \frac{\cos \phi}{\cos \theta} \end{bmatrix} \boldsymbol{\omega}^b \quad (2)$$

$$\left. \begin{aligned} \mathbf{a}_{\text{sensor}} &= \frac{1}{m} \mathbf{f}^b + \boldsymbol{\omega}^b \times (\boldsymbol{\omega}^b \times \mathbf{r}^b) + \mathbf{g}^b \\ \boldsymbol{\omega}_{\text{sensor}} &= \boldsymbol{\omega}^b + [\mathbf{J}^{-1} \boldsymbol{\tau}^b - \mathbf{J}^{-1} \boldsymbol{\omega}^b \times \mathbf{J} \boldsymbol{\omega}^b] \times \mathbf{r}^b \end{aligned} \right\} \quad (3)$$

Eq.(1) is the Newton-Euler equations in body coordinates. Eq.(2) represents the kinematic equation of rigid bodies that relates angular rate to euler angles and their derivatives. Eq.(3) is the measurement equations that relate measured data from accelerometers and gyroscopes to states parameters where  $\mathbf{r}^b$  is the vector pointing from C.G. to the point that an accelerometer is mounted relative to the body frame.

Because the body part is to be controlled at the horizontal position, changes in orientation are very small under normal operation. Equations (1)(2)(3) can be linearized about the horizontal and stable position.

$$\left. \begin{aligned} \ddot{\mathbf{x}}^b &\simeq \frac{1}{m} \mathbf{f}^b \\ \dot{\boldsymbol{\omega}}^b &\simeq \mathbf{J}^{-1} \boldsymbol{\tau}^b \end{aligned} \right\} \quad (4)$$

$$\dot{\boldsymbol{\Omega}} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \simeq \boldsymbol{\omega}^b \quad (5)$$

$$\mathbf{a}_{\text{sensor}} \simeq \frac{1}{m} \mathbf{f}^b - \mathbf{r}^b \times \mathbf{J}^{-1} \boldsymbol{\tau}^b + \mathbf{g}^b \quad (6)$$

By choosing  $\mathbf{q} = [\mathbf{x}^b \dot{\mathbf{x}}^b \boldsymbol{\Omega} \boldsymbol{\omega}^b]^T$  as states;  $\mathbf{u} = [\mathbf{f}^b \boldsymbol{\tau}^b]^T$  as inputs; and  $\mathbf{y} = [\mathbf{a}_{\text{sensor}} \boldsymbol{\omega}^b]^T$  as measurements, we can write the dynamic equations in state-space form.

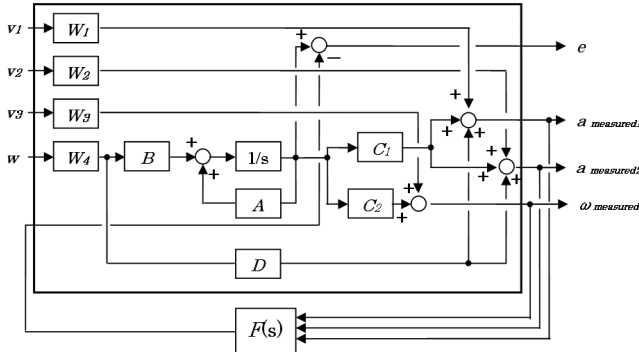


Fig.1 Generalized control system for designing the  $H_2$  optimal filter

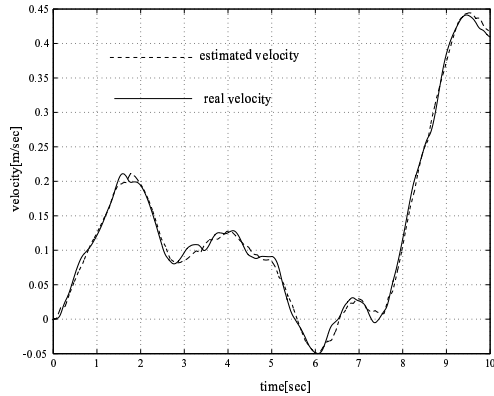


Fig.2 Comparison between estimated velocity and actual velocity from simulation

### 3. Sensor Unit

The sensor unit consists of two bi-axial gyroscopes (placed perpendicularly to form a tri-axial one), and two tri-axial accelerometers with different frequency response characteristics (one in low frequency region, the other in high frequency region). Sensor signals are filtered and amplified by analog op-amp circuits before entering A/D to the computer.

### 4. Filter Design

Fig.1 illustrates the block diagram used for designing the filter. The dynamic equations in state-space form are separated into blocks  $A, B, C_1, C_2$ , and  $D$ .  $W_4$  represents model of force-moment inputs. Acceleration and angular velocity of the system governed by force-moment input  $w$  are measured by two accelerometers and a gyroscope  $a_{\text{measured1}}, a_{\text{measured2}}, \omega_{\text{measured}}$ . Measurement is corrupted by noises  $v_1, v_2, v_3$  entering the system through the blocks representing noise models  $W_1, W_2$  and  $W_3$  respectively.  $F(s)$  is the filter that receives sensor signals and outputs estimates of the states. The difference between actual states and estimated states is designated as state errors  $e$ . The design policy is "design

a filter  $F(s)$  that minimizes the  $H_2$ -norm of the transfer matrix of inputs ( $v_1, v_2, v_3, w$ ) to the state errors  $e$ ". It is important to note that  $W_1, W_2, W_3$ , and  $W_4$  must be defined so that all conditions for existence of an optimal  $F(s)$  are satisfied<sup>6)</sup>. The MATLAB command `h2syn` is used to obtain the optimal filter.

### 5. Simulation results

We defined  $W_1, W_2$ , and  $W_3$  as complements in frequency response of each sensor (approximated by information from manufacturers) in order to emulate frequency characteristic of noises entering during measurement.  $W_4$  was modeled as a low-pass filter according to the assumption that low frequency inputs dominate motion of the body part. The effectiveness of the designed filter was tested by simulation on MATLAB and SIMULINK. In the simulation, we used the white noise of power 0.1 to generate noises  $v_1, v_2, v_3$  and of power 1 to generate the force-moment input signal  $w$ . The filter estimated states (euler angle, angular velocity, and position as well as velocity) from signals of angular velocity and acceleration corrupted by noises. The estimated velocity and actual velocity obtained from simulation is shown in figure Fig.2 as one example. Small error can be noticed.

### 6. Conclusions

We have developed a preliminary algorithm for the state estimator based on the linearized model. To implement it on the real humanoid robot, we need tuning some parameters in design process and refining models of inputs and disturbance to closely match the reality. Some trial and error and extensively testing on the real system are inevitable.

This research is supported by the Robot Brain Project of Core Research for Evolutional Science and Technology (CREST) of the Japan Science and Technology Corp.(JST).

### References

- 1) Marins et al: An Extended Kalman filter for Quaternion-based Orientation Estimation Using MARG Sensors, Proc. of the 2001 IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 2003-2011, 2001
- 2) Foxlin: Inertial Head-Tracker Sensor Fusion by a Complementary Separate-Bias Kalman Filter, Proc. of the 1996 Virtual Reality Annual International Symposium (VRAIS '96), pp. 185-194, 1996
- 3) Albert et al: A low-cost and Low-weight Attitude Estimation System for an Autonomous Helicopter, Proc. of the IEEE Int. Conf. on Intelligent Engineering Systems, (INES'97) pp.391-395, 1997
- 4) M.Koifman: Autonomously Aided Strapdown Attitude Reference System, Journal of Guidance and Control, vol. 14, No. 6, pp. 1164-1172, 1991.
- 5) M.Murray, Z.Li, S. Sastry: Robotic Manipulation CRC Press, 1994
- 6) T.Chen, B.Francis: Optimal Sampled-Data Control Systems, Communication and Control Engineering Series, Springer-Verlag, Berlin, 1995.