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Polynomial Design of Dynamics-based Information Processing System

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Abstract. For the development of the intelligent robot with many degree-of-freedom, the reduction of the whole body motion and the implementation of the brain-like information system is necessary. In this paper, we propose the reduction method of the whole body motion based on the singular value decomposition and design method of the brain-like information processing system using the nonlinear dynamics with polynomial configuration. By using the proposed method, we design the humanoid whole body motion that is caused by the input sensor signals.

1 Introduction

For the robot intelligence such as learning and searching, Neural Network and Generic Algorithm are mainly used as powerful tools. In these methods, a cost function is set and optimized. The focuses of conventional researches are how to define a cost function and how to optimize the cost function that defines robot motions based on sensor signals. The information processing in these methods is represented as the following sequence of modules.

1. Recognition of environment using sensor signal.
2. Selection of the robot motion.
3. Realization of the motion

Because this procedure is suitable for the recent computer (Neumann computer), the development of the faster processor enables the more sensing, selection and the faster calculation. And so far, many effective methods have been proposed. However, because this procedure reaches to the flame problem in the end, a new approach to the robot intelligence is expected.

Recently, the approach that the intelligence is the result of cooperation between the robot body dynamics and the environments has received attention. For example, in Subsumption Architecture[1], some modules are connected each other, which yields robot motion based on the sensor feedback through the environment. This method aims at producing the robot intelligence using interaction of the robot body and environment.

Since Freeman showed the dynamical phenomenon in the rabbit olfactory such as entrainment and chaos phenomenon[2–4], some researchers have tried to realize the robot intelligence using dynamical phenomena. Nakamura and

Sekiguchi developed the information processing system using chaotic dynamics[5,6]. They designed a control algorithm for a mobile robot using entrainment and synchronization of the robot dynamics and environment dynamics based on Arnold differential equation.

In this paper, we propose the dynamics-based brain-like information processing system using dynamical phenomena such as entrainment and detrainment to attractors, and give a design strategy of the dynamics-based information processing system that handles humanoid whole body motions. The nonlinear dynamics with polynomial representation memorizes, generates and transits humanoid whole body motions based on the entrainment and detrainment phenomenon . Some design methods of the nonlinear dynamics that has an attractor using the Lyapunov function have been proposed [7]∼[9]. In our method, the dynamics is defined as the vector field in N dimensional space and it has an attractor to some closed curved lines. By changing the vector field configuration, the dynamics behavior is changeable.

On the other hand, because the whole body motions of humanoid robots consist of many joint angles data, it requires much computational quantity to deal with those motions. In this paper, we propose a reduction and symbolization method of the whole body motion based on the principal component analysis[10] using singular value decomposition.

2 Dynamics and whole body motion

The human does not remember the time sequence pattern of the joint angle respect to a whole body motion. The motions are symbolized and selected appropriately based on the internal state and input sensor signals. We show this process and phenomenon corresponding to the dynamics. Consider the following discrete time dynamical equation.

$$\boldsymbol{x}[k+1] = \boldsymbol{x}[k] + \boldsymbol{g}(\boldsymbol{u}[k], \boldsymbol{x}[k]) \tag{1}$$

$\boldsymbol{x} \in \boldsymbol{R}^N$ is the state vector and $\boldsymbol{u} \in \boldsymbol{R}^L$ is the input signal. $\boldsymbol{x}[k]$ moves in the N dimensional space. If this dynamics has attractor to one closed curved line C, $\boldsymbol{x}[k]$ is entrained to C according to $k \to \infty$ with an initial condition $\boldsymbol{x}_0$ and $\boldsymbol{u}[k] = 0$. Suppose that the C represents the time sequence data of a motion M. The time sequence data of $\boldsymbol{x}[k]$ yields that of humanoid joint angles, which means the dynamics memorizes and generates the humanoid whole body motion.

Suppose the dynamics in equation (1) has some attractors and transits to each attractors by the input signal $\boldsymbol{u}[k]$, which means the motion transition of the humanoid robot based on the input sensor signal.

In this paper, we design the brain-like information processing by using nonlinear dynamics that memorizes and generate the whole body motion as an attractor.

3 Motion reduction and symbolization

For the design of a dynamics that handles a humanoid whole body motion, a reduction is necessary to decrease the calculation. By the mapping function and its inverse function, the whole body motion is reduced to the symbol (reduced dimensional data) and restored from symbol. Consider the humanoid robot with n degrees-of-freedom. The humanoid motion data Y is defined as follows using time sequence data $y_i[k]$.

$$Y = \begin{bmatrix} y_1[1] & y_1[2] & \cdots & y_1[m] \\ y_2[1] & y_2[2] & \cdots & y_2[m] \\ \vdots & \vdots & & \vdots \\ y_n[1] & y_n[2] & \cdots & y_n[m] \end{bmatrix} \tag{2}$$

For example, $y_i[k]$ means angles of each joint. By the singular value decomposition of Y

$$Y = USV^T \tag{3}$$

$$U = \left[U_1 \middle| U_2 \right] \tag{4}$$

$$S = \left[\begin{array}{c|c} S_1 & \\ \hline & S_2 \end{array} \right] \tag{5}$$

$$S_1 = \mathrm{diag}\left\{ s_1 \; s_2 \; \cdots \; s_r \right\} \tag{6}$$

$$S_2 = \mathrm{diag}\left\{ s_{r+1} \; s_{r+2} \; \cdots \; s_n \right\} \tag{7}$$

$$V^T = \left[\begin{array}{c} V_1^T \\ \hline V_2^T \end{array} \right] \tag{8}$$

if $s_r \gg s_{r+1}$ is satisfied, Y is reduced to r dimensional motion V_1^T as follows.

$$Y = FV_1^T \tag{9}$$

$$F = U_1 S_1 \tag{10}$$

Here, $U_1 \in \boldsymbol{R}^{n\times r}$, $V_1^T \in \boldsymbol{R}^{r\times m}$. F is the mapping function from the symbol to whole body motion. This result shows that

1. The whole body motion Y that represents a curved line in n dimensional space, is symbolized by a reduced dimensional curved line V_1^T in r dimensional space. If $y_i[k]$ $(i = 1, \cdots n,\; k = 1, \cdots m)$ is the periodic sequence, Y and V_1^T mean the closed curved lines M and C respectively.
2. By checking the singular value s_i $(i = 1, 2, \cdots, n)$, the appropriate r can be selected.
3. The first column vector of V_1 is the first principal component of the motion Y, the second column vector is the second principal component.
4. The inverse function of F is $S_1^{-1}U_1^T$

4 Design of dynamics-based information processing system

4.1 Design of the nonlinear dynamics

Consider the reduced whole body motion in N dimensional space. The time sequence data of each degree-of-freedom is set as $\xi_i[k]$ ($i = 1, \cdots N,\ k = 1, \cdots m$). Assuming the time sequence data is periodic, $\boldsymbol{\xi}[k]$

$$\boldsymbol{\xi}[k] = \left[\xi_1[k]\ \xi_2[k] \cdots \xi_N[k]\right]^T \tag{11}$$

consists the closed curved line Ξ in the N dimensional space.

$$\Xi = \left[\boldsymbol{\xi}[1]\ \boldsymbol{\xi}[2] \cdots \boldsymbol{\xi}[m]\ \boldsymbol{\xi}[1]\right] \tag{12}$$

Figure 1 shows the simple example of $N = 2$. Two time sequences $\xi_1[k],\ \xi_2[k]$

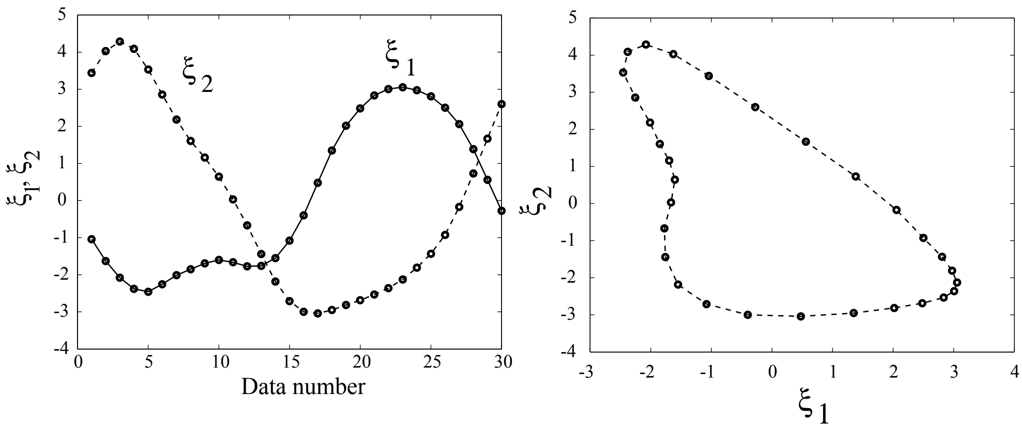

Fig. 1. Time sequence data and closed curved line

construct the closed curved line in the 2 dimensional space. In this section, we design a nonlinear dynamics $\mathcal{D}$ that has an attracter to this closed curved line with the following formulation.

$$\mathcal{D}\ :\ \boldsymbol{x}[k+1] = \boldsymbol{x}[k] + \boldsymbol{f}(\boldsymbol{x}[k]) \tag{13}$$

The design algorism is as follows.

Step 1 Draw the closed curved line C in equation (12) in the N dimensional space.

Step 2 Define the vector field $\boldsymbol{f}(\boldsymbol{\eta}_i)$ on the point $\boldsymbol{\eta}_i$ which is in domain D in the N dimensional space according to the following algorism (refer to figure 2).

$$\boldsymbol{f}(\boldsymbol{\eta}_i) = (\boldsymbol{\xi}_\eta[k+1] - \boldsymbol{\xi}_\eta[k]) + \boldsymbol{\gamma}_i[k] \tag{14}$$

$$\boldsymbol{\xi}_\eta[k] = \arg\min_{\boldsymbol{\xi}[k]} ||\boldsymbol{\eta}_i - \boldsymbol{\xi}[k]|| \tag{15}$$

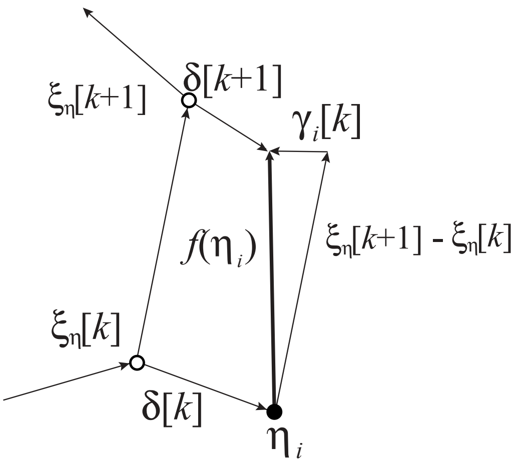

Fig. 2. Vector definition

Because $\boldsymbol{\eta}_i = \boldsymbol{\xi}_\eta[k] + \boldsymbol{\delta}[k]$, the sufficiency of that the closed curved line becomes the attractor is to satisfy the following inequarity.

$$||\boldsymbol{\delta}[k] + \boldsymbol{\gamma}_i[k]|| < ||\boldsymbol{\delta}[k+1]|| \tag{16}$$

By satisfing this condition, $\boldsymbol{\delta}[k] \rightarrow 0$ at $k \rightarrow \infty$.

Step 3 On the points $\boldsymbol{\eta}_1,\ \boldsymbol{\eta}_2,\ \cdots,\ \boldsymbol{\eta}_m$ in the domain D, define the vectors $\boldsymbol{f}(\boldsymbol{\eta}_1),\ \boldsymbol{f}(\boldsymbol{\eta}_2),\ \cdots,\ \boldsymbol{f}(\boldsymbol{\eta}_m)$ shown in figure 3

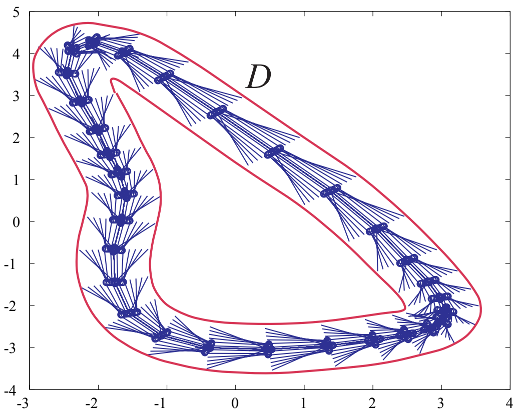

Fig. 3. Definition of the vector field

Step 4 Obtain the nonlinear function $\boldsymbol{f}(\boldsymbol{x}[k])$ that approximate $\boldsymbol{f(\eta_i)}$ by using the polynomial approximation of $\boldsymbol{x}_i$ as follows.

$$\boldsymbol{f}(\boldsymbol{\eta}_i) = \sum_{P=0}^{l} \sum_{\substack{p_1,\cdots,p_n \\ \sum p_k = P \\ p_k\ :\ \text{positive integer}}}^{l} a_{(p_1 p_2 \cdots p_n)} \prod_{j=1}^{n} \eta_{ij}^{p_j} \tag{17}$$

$$\boldsymbol{\eta}_i = \left[\eta_{i1}\ \eta_{i2}\ \cdots\ \eta_{iN} \right]^T \tag{18}$$

a_{ij}^{pq} are constants. In the case of N=2 and ℓ=3, equation (17) is represented as follows.

$$\begin{aligned}\boldsymbol{f}(\boldsymbol{\eta}_i) &= a_{(30)}\eta_{i1}^3 + a_{(21)}\eta_{i1}^2\eta_{i2} + a_{(12)}\eta_{i1}\eta_{i2}^2 + a_{(03)}\eta_{i2}^3 \\ &+ a_{(20)}\eta_{i1}^2 + a_{(11)}\eta_{i1}\eta_{i2} + a_{(02)}\eta_{i2}^2 \\ &+ a_{(10)}\eta_{i1} + a_{(01)}\eta_{i2} + a_{(00)}\end{aligned} \tag{19}$$

It is easy to calculate $\boldsymbol{f}(\boldsymbol{x}[k])$ by the least square method as follows.

$$\Phi(a_{pq}^{ij}) = FH^{\#} \tag{20}$$

$$F = \left[\, \boldsymbol{f}(\boldsymbol{\eta}_1)\ \boldsymbol{f}(\boldsymbol{\eta}_2)\ \cdots\ \boldsymbol{f}(\boldsymbol{\eta}_m) \,\right] \tag{21}$$

$$H = \begin{bmatrix} \eta_{11}^{\ell} & \eta_{21}^{\ell} & \cdots & \eta_{m1}^{\ell} \\ \vdots & & & \\ \eta_{1N}^{\ell} & \eta_{2N}^{\ell} & \cdots & \eta_{mN}^{\ell} \\ \eta_{11}^{\ell-1}\eta_{12} & \eta_{21}^{\ell-1}\eta_{22} & \cdots & \eta_{m1}^{\ell-1}\eta_{m2} \\ \vdots & & & \\ 1 & 1 & \cdots & 1 \end{bmatrix} \tag{22}$$

Φ is a constant matrix. The domain D is the basin, which means the state $\boldsymbol{x}[k]$ inside D is entrained to C at $k \to \infty$. If Φ gives a right approximation of the defined vector field, we obtain the nonlinear dynamics that has an attractor to the closed curved line C. Figure 4 shows the motion of the designed nonlinear

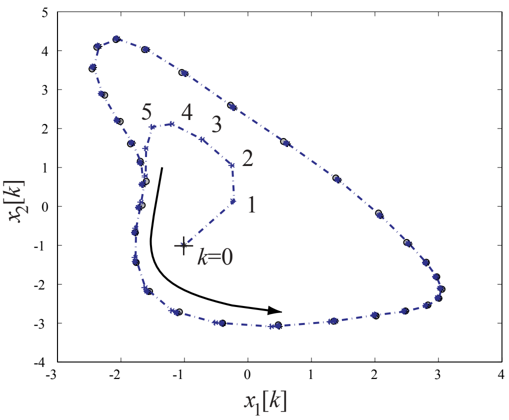

Fig. 4. Motion of the nonlinear dynamics

dynamics. '+' means the initial condition of $\boldsymbol{x}[0]$. It is entrained to C. This result shows that the designed dynamics memorizes the time sequence data $\boldsymbol{\xi}[k]$ $(k = 1,\ 2,\ \cdots\ m)$ and generate the whole body motion.

4.2 Set of the input signal

The designed dynamics $\mathcal{D}$ in equation (13) moves autonomously. From the initial state condition $\boldsymbol{x}[0]$ in the domain D, it converges to C. In this section, we set the input signal to the designed dynamics. By the input signal, the nonlinear dynamics is entrained or detrained to the attractor, which means this dynamics has the following functions.

Association of whole body motion By observing a part of whole body motion (for example, motion of arm), the robot associate the rest of motions (for example, motion of leg and body).

Module component The dynamics-based information processing system becomes a mdule component with input and output signal, which enables to design the dynamics network.

Recognition of self motion By the comparison of input signal and its estimation, the robot recognizes the observed motion or self motion. This is discussed in the later section.

By adding another time sequence $\boldsymbol{u}[k]$ $(\in \boldsymbol{R}^L,\ k = 1,\ 2,\ \cdots,\ M)$, the dynamics is represented as follows.

$$\widehat{\mathcal{D}} \ :\ \boldsymbol{x}[k+1] = \boldsymbol{x}[k] + \boldsymbol{g}\left(\boldsymbol{u}[k], \boldsymbol{x}[k]\right) \tag{23}$$

By changing Ξ in equation (12) as

$$\Xi = \begin{bmatrix} \boldsymbol{\mu}[1] & \boldsymbol{\mu}[2] & \cdots & \boldsymbol{\mu}[M] & \boldsymbol{\mu}[1] \\ \boldsymbol{\xi}[1] & \boldsymbol{\xi}[2] & \cdots & \boldsymbol{\xi}[M] & \boldsymbol{\xi}[1] \end{bmatrix} \tag{24}$$

$$\boldsymbol{\mu}[k] \ :\ \text{given}$$

$\boldsymbol{g}(\boldsymbol{u}[k], \boldsymbol{x}[k])$ is calculated by the same algorism as $\boldsymbol{f}(\boldsymbol{x}[k])$. Figure 5 shows the motion of the dynamics with 1 dimensional input $u[k]$. The input $u[k]$ is

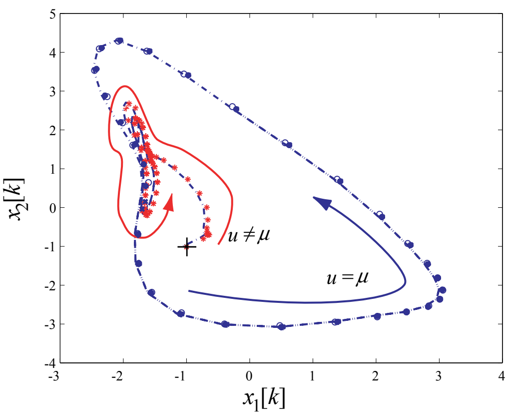

Fig. 5. Motion of the nonlinear dynamics with input signal

changed from one signal to $\mu[k]$ in time $k = \widehat{k}$. The dynamics is entrained to C

after going around in the space according to the change of input. This result means that the input signal changes the structure of the nonlinear dynamics by changing the vector field $\boldsymbol{g}(\boldsymbol{u}[k], \boldsymbol{x}[k])$, and entrains the dynamics to C.

4.3 Dynamics with multi attractors

We modify the dynamics $\widehat{D}$ so that it has some attractors and transits to each attractors. The nonlinear dynamics is re-written as follows.

$$\widehat{\mathcal{D}}^w \;:\; \boldsymbol{x}[k+1] = \boldsymbol{x}[k] + w(\boldsymbol{x}[k])\boldsymbol{g}\left(\boldsymbol{u}[k], \boldsymbol{x}[k]\right) \tag{25}$$

$w(\boldsymbol{x}[k])$ is the weighting function defined as follows.

$$w(\boldsymbol{x}[k]) = 1 - \frac{1}{1 + \exp\{a(\omega(\boldsymbol{x}[k]) - 1)\}} \tag{26}$$

$$\omega(\boldsymbol{x}[k]) = (\boldsymbol{x}^T[k] - X_0^T)Q(\boldsymbol{x}[k] - X_0) \tag{27}$$

a is a constant. Q and X_0 define the following ellipsoid E.

$$(\boldsymbol{x}^T[k] - X_0^T)Q(\boldsymbol{x}^T[k] - X_0) = 1 \tag{28}$$

X_0 is the center of the ellipsoid, Q is the positive definite matrix. Equation (26) means that if $\boldsymbol{x}[k]$ is inside the ellipsoid in equation (28), the weighting function is 1. Based on the dynamics $\widehat{\mathcal{D}}_i^w$ $(i = 1,\ 2,\ \cdots)$, we design the nonlinear dynamics $\widetilde{\mathcal{D}}$ which has some attractors as follows.

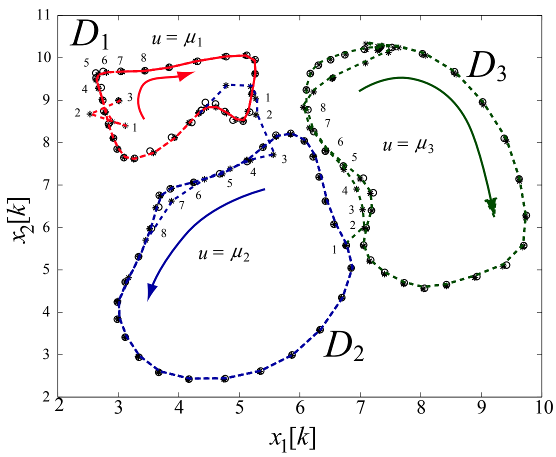

Fig. 6. Designed three attractors and dynamics

$$\widetilde{\mathcal{D}} \;:\; \boldsymbol{x}[k+1] = \boldsymbol{x}[k] + \sum_i w_i(\boldsymbol{x}[k])\boldsymbol{g}_i(\boldsymbol{u}[k], \boldsymbol{x}[k]) \tag{29}$$

This configuration means that the vector field that defines an attractor is effective only in the ellipsoid. One vector field defined by $\boldsymbol{g}_i(\boldsymbol{u}[k], \boldsymbol{x}[k])$ dose not have influence to other attractors. Because the vector fields is defined by the sum of $\boldsymbol{g}_i(\boldsymbol{u}[k], \boldsymbol{x}[k])$ surrounded by the ellipsoid E_i, it is easy to add the new attractor. The state $\boldsymbol{x}[k]$ moves to some attractors in term of the input signal $\boldsymbol{u}[k]$. Figure 6 shows the designed dynamics that has three attractors in the 2 dimensional space. By the 1 dimensional input $\boldsymbol{u}[k]$, the dynamics moves in the space attracted to the closed curved lines.

4.4 Recognition of self motion

The designed dynamics in equation (23) is modified as follows.

$$\bar{\mathcal{D}} \ : \ \begin{bmatrix} \hat{\boldsymbol{u}}[k+1] \\ \boldsymbol{x}[k+1] \end{bmatrix} = \begin{bmatrix} \boldsymbol{u}[k] \\ \boldsymbol{x}[k] \end{bmatrix} + \boldsymbol{h}(X[k]) \tag{30}$$

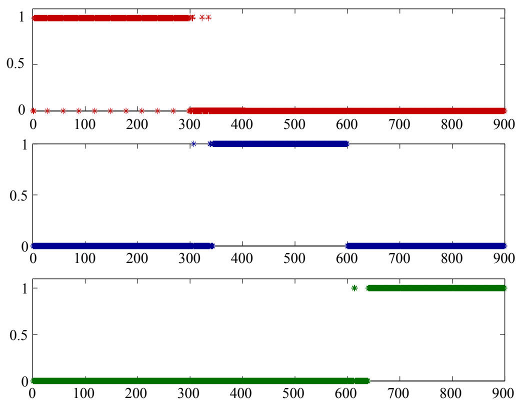

Fig. 7. Recognition of the input signal

The vector field $\boldsymbol{h}$ is calculate by the same ways as $\boldsymbol{f}$ and $\boldsymbol{g}$. Because $\hat{\boldsymbol{u}}[k+1]$ is the prediction of the $\boldsymbol{u}[k+1]$, this dynamics recognizes which closed curved line the state vector is attracted by comparing $\boldsymbol{u}[k+1]$ and $\hat{\boldsymbol{u}}[k+1]$. Figure 7 shows the recognition index of the attraction. The vertical axis is the recognition index (RI) that is defined

$$RI_i = \begin{cases} 1 \ (||\hat{\boldsymbol{u}}[k+1] - \boldsymbol{u}[k+1]|| \geq \alpha) \\ 0 \ (||\hat{\boldsymbol{u}}[k+1] - \boldsymbol{u}[k+1]|| < \alpha) \end{cases} \tag{31}$$

The upper figure, the middle figure and the lower figure show the recognition of $\boldsymbol{u}_i[k]$ (correspond to $\mathcal{D}_i$, $i = 1, \ 2, \ 3$) respectively. By checking RI_i, the humanoid recognizes which motion it takes, which means the recognition of self motion.

5 Generation of the whole body motion

5.1 Whole body motion of the humanoid robot

In this section, we design the humanoid whole body motion using the dynamics based information processing system. Figure 8 shows the humanoid robot (FUJITSU Humanoid HOAP-1) that has 20 degree of freedom. We design

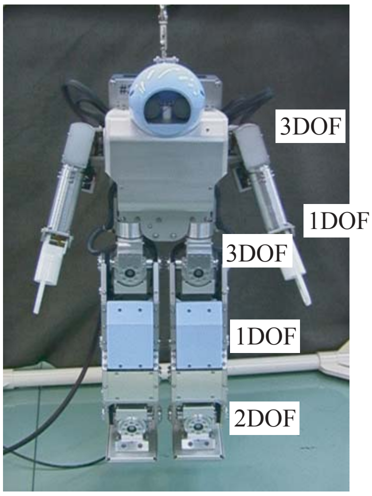

Fig. 8. Humanoid robot HOAP-1

the "walk" motion and "squat" motion. Figure 9 shows the original motion. This humanoid robot is not grounded. Because the dynamic based information processing system yields only the time sequence of the joint angle, the feedback controller that stabilizes each motions should be implemented.

5.2 Design of the dynamics

From the "walk" motion $Y_w \in \boldsymbol{R}^{20}$ and "squat" motion $Y_s \in \boldsymbol{R}^{20}$, we obtain the reduced motions $V_w^T \in \boldsymbol{R}^3$, $V_s^T \in \boldsymbol{R}^3$ in three dimensional space.

Based on the reduced motion, we design the dynamics based on the following equation

$$\boldsymbol{x}[k+1] = \boldsymbol{x}[k] + \sum_{i=w,s} w_i(\boldsymbol{x}[k])\boldsymbol{f}_i(\boldsymbol{x}[k]) + \sum_{i=w,s} K_i O_i(\boldsymbol{x}[k]) \quad (32)$$

$$O_i(\boldsymbol{x}[k]) = \delta(X_i^c - \boldsymbol{x}[k]) \quad (33)$$

By changing K_w and K_s, the humanoid transits its motion. δ is constant. X_w^c and X_s^c mean the center of reduced closed curved line "walk" and "squat" respectively. Figure 10 shows the motion of the dynamics. From the initial

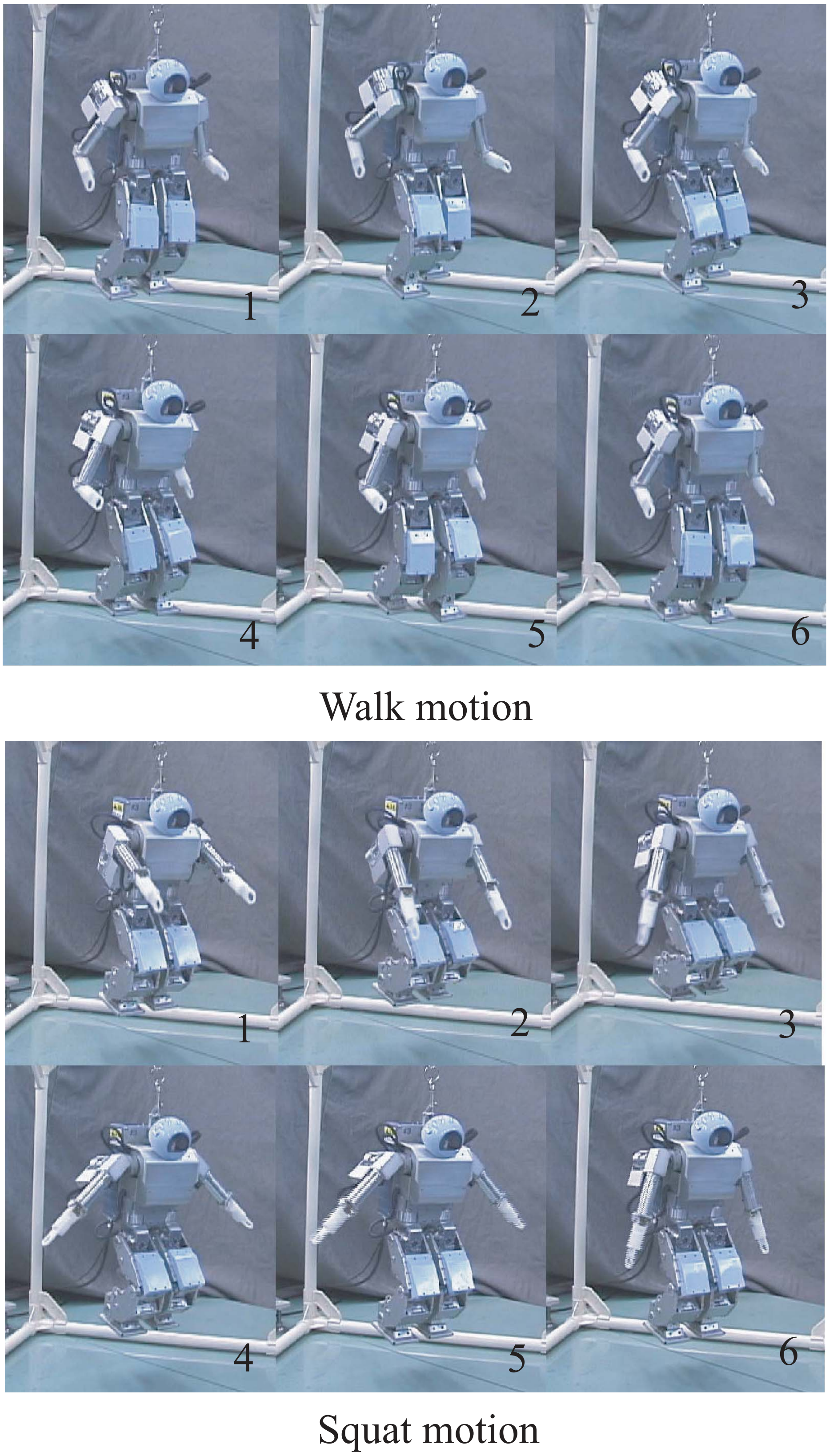

Fig. 9. Humanoid motion

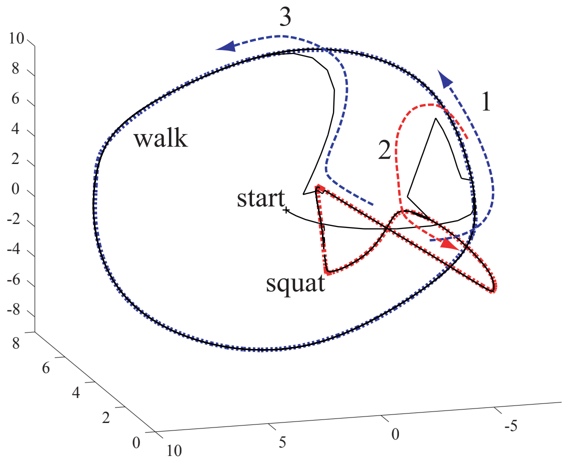

Fig. 10. Motion of the dynamics

position, the dynamics is entrained to the walk motion (arrow 1), entrained to squat motion (arrow 2) and finally entrained to walk motion again (arrow 3).

5.3 Motion of the humanoid robot

Figure 11 shows the generated humanoid motion. Because while the dynamics is attracted to the closed curved line, the humanoid motion is as same as figure 9, and only the transition motions are shown. The continuous transition is generated.

6 Conclusions

In this paper, we propose the motion reduction method and the brain-like information processing that realizes the memorization and generation of the humanoid whole body motion using the nonlinear dynamics with the polynomial configuration. The results of this paper are as follows.

1. We propose the motion reduction method using the principal component analysis based on singular value decomposition.
2. We propose the design method of the nonlinear dynamics that has an attractor to the N dimensional closed curved line with polynomial configuration.
3. Using the proposed method, the whole body humanoid motion is generated.

The proposed method is a design method of the nonlinear dynamics that has some attractors to any curved line in N dimensional space. The hierarchical structure and dynamics network gives a new approach to the robot intelligence. How to design and how to connect the network of the dynamics are future problems.

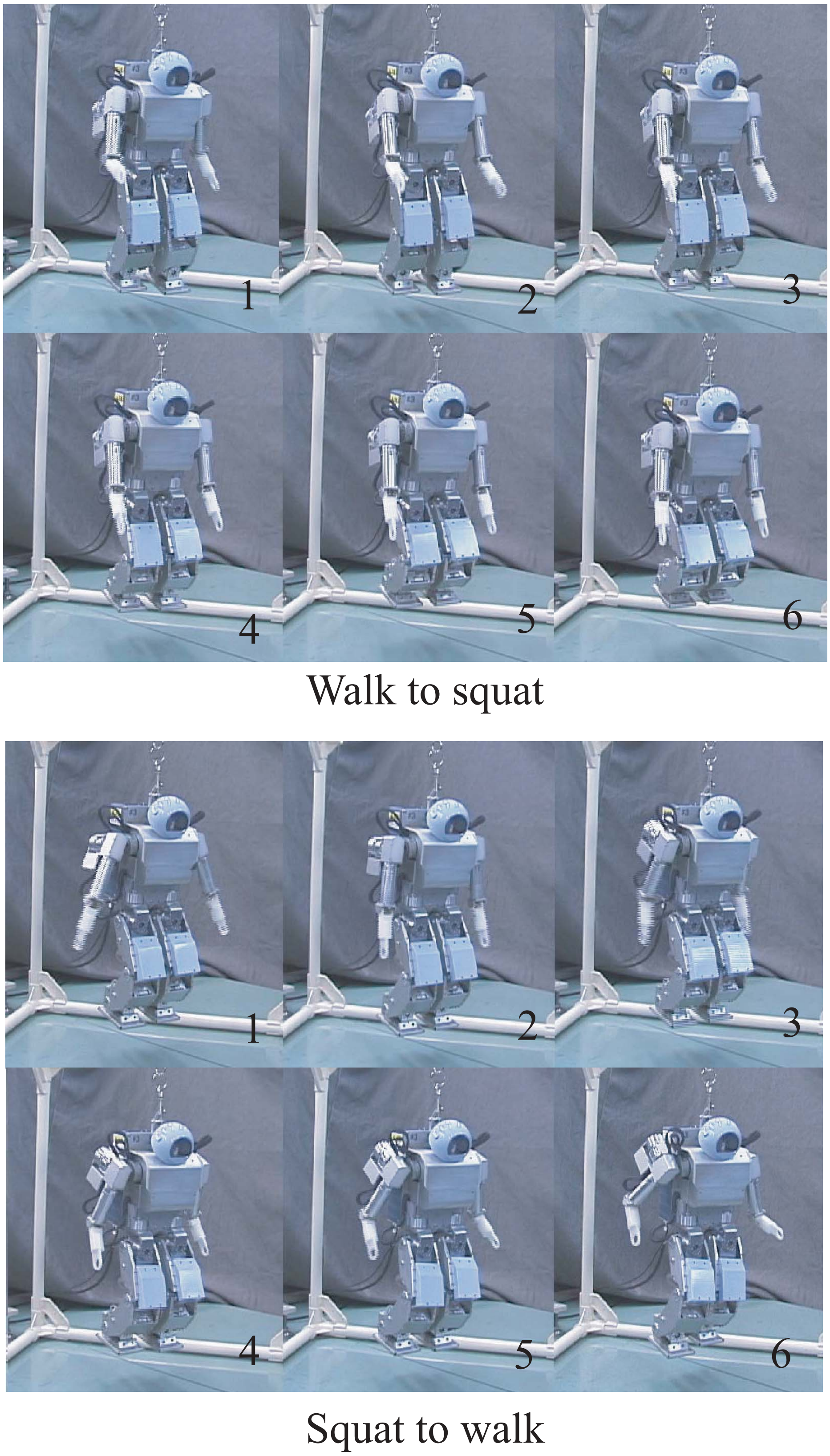

Fig. 11. Motion transintion of the humanoid robot

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